# In-sector compressive direction-of-arrival estimation with a switched receive array

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***Abstract*—Direction-of-arrival (DoA) estimation is a key function in 5G radios, radars, and sonars. While large arrays lead to high-resolution DoA estimates, their implementation with fully digital receive architectures incurs significant power consumption. This paper demonstrates DoA estimation with a low-power switched receive array, which consumes less power than a fully digital array. The DoA is estimated in two stages. First, the sector in which the source lies is estimated by mechanically steering a wide beam. Then, the DoA within the identified sector is estimated electronically using our switched receive array. We formulate an integer program to optimize the configuration of the switches in the receiver. Our optimized configuration results in low-aliasing artifacts within the identified sector. We build a custom ultrasound receive array testbed and demonstrate DoA estimation with our optimized switch configuration.**

**Video:** https://www.youtube.com/watch?v=-2LHcuZD1S0

## I. Background and system model

Switched arrays are a low-power and low-complexity alternative to fully digital arrays, making them well suited for energy-critical applications. The low-power consumption is due to fewer analog-to-digital converters (ADCs) in a switched array than its fully digital counterpart, as illustrated in Fig. 1a. However, estimating the DoA with a switched array is challenging, as it captures a compressed spatial representation of the incoming signal. To address this problem, we adopt hierarchical DoA estimation, which is commonly used in 5G. In this approach, the sector containing the ultrasound sound source is first identified as shown in Fig. 1b, followed by finer estimation of the DoA within that sector. Our contribution is in optimizing the switches selected, equivalently the sparse array, for DoA estimation within the identified sector.

**System model:** We consider a uniform linear array (ULA) with $N$ microphones at the receiver (RX). The inter-element spacing at the RX is $d$. We focus on estimating the DoA associated with a single sound source transmitter (TX) operating at a wavelength of $\lambda$. We use $\mathbf{h} \in \mathbb{C}^{N\times 1}$ to denote the impinging spatial signal at the RX, also called as the channel between the TX and the RX. Considering a line-of-sight channel with a scaling $\alpha$ and a direction-of-arrival $\theta_o$,

$$\mathbf{h} = \alpha[1, e^{\mathrm{j}2\pi\frac{d}{\lambda}\sin\theta_o}, ..., e^{\mathrm{j}2\pi(N-1)\frac{d}{\lambda}\sin\theta_o}]^T. \tag{1}$$

We denote $M$ as the number of ADCs. In a switched array, $M < N$ to limit the power consumption. Therefore, only $M$ microphone measurements can be acquired simultaneously in our system. The choice of these channels is determined by the programmable switches, as shown in Fig. 1(a).

We use $\mathcal{S}$ to denote the indices of the microphone channels acquired, e.g., $\mathcal{S} = \{1, 2, 5, 7\}$ for $M = 4$. To model the received measurements, we define an $N \times 1$ binary vector $\mathbf{s}$ that is one at the locations in $\mathcal{S}$ and zero otherwise. For example, $\mathbf{s} = [1, 1, 0, 0, 1, 0, 1, 0]$ when $\mathcal{S} = \{1, 2, 5, 7\}$. The acquired measurement vector

$$\mathbf{y} = \mathbf{s} \odot \mathbf{h} + \mathbf{s} \odot \mathbf{v}, \tag{2}$$

where $\odot$ is the Hadamard product and $\mathbf{v}$ is additive Gaussian noise. Here, $y[m] = h[m]+v[m]$ for $m \in \mathcal{S}$ and is 0 otherwise.

**Sector search**: A sector typically consists of a contiguous range of directions, as illustrated in Fig. 1b. Although sector search can be implemented electronically, we adopt a simple mechanical steering approach to identify the sector containing the TX. Specifically, we constrain the RX's field of view to a sector of width $\Delta$ using two physical barriers, as shown in Fig. 1c. We then rotate the RX array in discrete steps, computing the sum of received signal magnitudes, $\sum |y[n]|$, at each orientation. The sector corresponding to the maximum sum is identified as the one containing the TX. We denote the set of angles within this sector as $\Theta_{\mathrm{tx}}$, e.g., $\Theta_{\mathrm{tx}} = \{\theta : |\theta| \leq 25°\}$.

## II. In-sector DoA estimation

Our goal is to estimate the DoA while leveraging information that the DoA is within $\Theta_{\mathrm{tx}}$. To this end, we optimize the switch configuration in $\mathbf{s}$ to suppress aliasing artifacts within the TX's sector $\Theta_{\mathrm{tx}}$, where accurate estimation is critical, while permitting significant aliases outside this sector.

**Aliasing:** Acquiring few spatial measurements of $\mathbf{h}$, i.e., using a sparse $\mathbf{s}$, results in aliasing artifacts in the angle-domain. This angle-domain representation of the acquired channel is given by the discrete-time Fourier transform (DTFT) of $\mathbf{y}$, denoted by $Y(e^{\mathrm{j}\Omega})$. A matched filter-based DoA estimate $\hat{\theta}$ is the angle that maximizes the DTFT, i.e.,

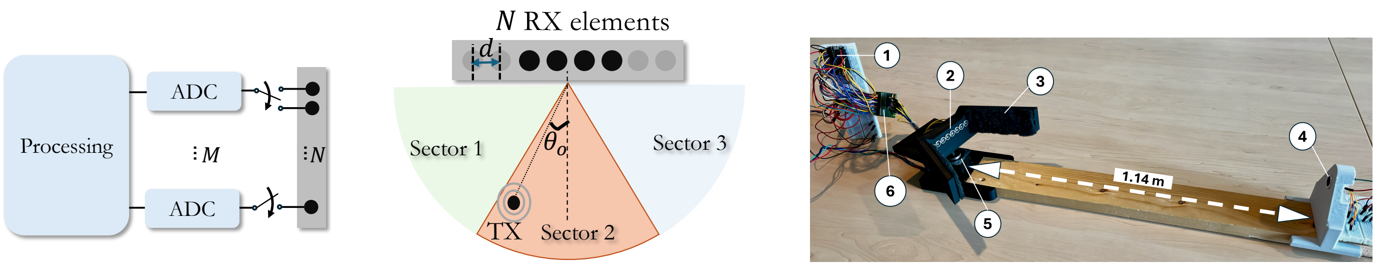
$$\hat{\theta} = \arg\max_{\theta\in\Theta_{\mathrm{tx}}} \left|Y(e^{\mathrm{j}2\pi d \sin\theta/\lambda})\right|. \tag{3}$$

To explain aliasing, we use the property that multiplication of vectors in (2) is equivalent to convolution of their DTFTs, i.e.,

$$Y(e^{\mathrm{j}\Omega}) = \frac{1}{2\pi}\int_{2\pi} S(e^{\mathrm{j}\phi})H(e^{\mathrm{j}(\Omega-\phi)})d\phi + V_s(e^{\mathrm{j}\Omega}), \tag{4}$$

where $S(e^{\mathrm{j}\Omega})$, $H(e^{\mathrm{j}\Omega})$ and $V_s(e^{\mathrm{j}\Omega})$ are the DTFTs of $\mathbf{s}$, $\mathbf{h}$ and $\mathbf{s} \odot \mathbf{v}$ respectively.

We explain aliasing due to a switched receive array using (4). We first consider the extreme case of $M = N$ which results in $\mathbf{s} = \mathbf{1}_N$, equivalently its DTFT $S(e^{\mathrm{j}\Omega})$ is a Dirichlet's




(a) System overview (b) Sector search (c) Experimental setup

Fig. 1: Fig 1a shows a switched receive array, where $M$ switches are used to select which of the $N$ microphones are connected to the $M$ ADCs. The RX array is first mechanically steered to scan different sectors, as shown in Fig 1b. Then, the DoA within the sector is electronically determined. Our testbed in Fig 1c comprises: 1. Teensy 2. RX array 3. Acoustic barrier 4. TX 5. Stepper motor 6. ADC.

sinc function. In this case, the result of the convolution in (4) is also a Dirichlet's sinc with its maximum at $\Omega_o = 2\pi d \sin\theta_o/\lambda$. As $M < N$ in a switched array, the switch configuration vector $\mathbf{s}$ is sparse with $M$ ones, due to which $S(e^{j\Omega})$ is no longer a Dirichlet's sinc. When $M < N$, $S(e^{j\Omega})$ contains significant grating lobes as shown in Fig. 2. These grating lobes, called aliasing artifacts, are determined by $\mathbf{s}$. Such artifacts lead to false alarms and misdetection at low signal-to-noise ratio. For example, a source at 0 degrees results in a significant matched-filter artifact at 14 degrees in Fig. 2a, which makes it hard to detect another source at 14 degrees. As the RX receives signals only from the identified sector, it is important to design an $\mathbf{s}$ that minimizes the grating lobes of $S(e^{j\Omega})$ only over $\Theta_{\mathrm{tx}}$. Any artifacts outside this sector do not matter as the receiver listens and detects transmissions only from $\Theta_{\mathrm{tx}}$.

**Proposed optimization of s:** For a switched receive array with $M$ ADCs, the number of ones in $\mathbf{s}$ should be $M$, i.e., $\mathbf{1}^T\mathbf{s} = M$. Under this constraint, our method finds the best $\mathbf{s}$ that results in the lowest aliasing artifacts over the sector $\Theta_{\mathrm{tx}}$. Our optimization problem can be formally stated as

$$\min_{\mathbf{s}\in\{0,1\}^N} \max_{\theta\in\Theta_{\mathrm{tx}}} |S(e^{j2\pi d\sin\theta/\lambda})|, \quad \text{s.t.} \quad \mathbf{1}^T\mathbf{s} = M, \tag{5}$$

The problem in (5) is non-convex and a bruteforce search is intractable for large arrays. To solve for $\mathbf{s}$ in (5), we discretize $\theta$ and define an auxiliary variable $\gamma \geq 0$ to convert (5) into

$$\begin{aligned} \min_{\{\gamma,\mathbf{s}\}} \gamma \quad \text{s.t.} \quad & \max_{\theta\in\Theta_{\mathrm{tx}}^{\mathrm{disc}}} |S(e^{j2\pi d\sin\theta/\lambda})| \leq \gamma, \\ & \mathbf{s}\in\{0,1\}^N, \ \mathbf{1}_N^{\mathrm{T}}\mathbf{s} = M, \end{aligned} \tag{6}$$

We solve the integer program in (6) using the approach in [1]. The optimized switch configuration $\mathbf{s}_{\mathrm{opt}}$ is then used to select the $M$ channels from the microphone array.

## III. Hardware and Experimental Results

We built a custom 40 KHz ultrasound system with $N = 8$ RX microphones and a single speaker TX. The inter-element spacing was $d = 1.166\lambda$ due to the microphone size. The distance between the TX and the RX was 1.14 m, which is in the far-field at 40 KHz. The receiver array, shown in Fig. 1c, acquires time-domain samples at about 168 KHz. This array is mounted on a stepper motor for mechanically steering over three predefined sectors. The acquired samples are then used to measure the phase shifts in (1) using Teensy 4.1. The switched receiver measurement model in (2) is emulated by subsampling the acquired spatial channel measurements according to $\mathbf{s}$.

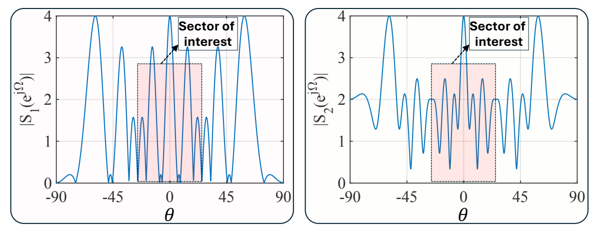



Fig. 2: Aliasing artifacts in matched filtering for two switch configurations $\mathbf{s}_1 = [1, 0, 0, 1, 1, 0, 0, 1]^{\mathrm{T}}$ and $\mathbf{s}_2 = [1, 1, 0, 1, 0, 0, 0, 1]^{\mathrm{T}}$. Here, $\Omega = 2\pi d \sin\theta/\lambda$, $M = 4$ and $N = 8$. We observe that $\mathbf{s}_2$ results in lower aliasing artifacts than $\mathbf{s}_1$ in the identified sector.

We use $\Delta_{\max} \approx 50°$ as the sector width in Fig. 1b. This limit is derived from $2\pi d|\sin(\Delta\max/2)|/\lambda \leq \pi$, which ensures low grating lobes for $M = N$. The RX array is mechanically steered for three distinct central angles: $-50°$, $0°$ and $50°$, thereby covering a $150°$ field of view. For a source placed at $\theta_o = 15°$, our setup correctly identifies it as belonging to sector 2 in Fig. 1b. Next, we sketch the matched filtered output in (4) for the switch configurations in Fig. 2. We observe from Fig. 3 that our optimized configuration $\mathbf{s}_{\mathrm{opt}}$ successfully estimates the DoA, while the random choice $\mathbf{s}_1$ does not. Finally, we artificially add noise $\mathbf{n}$ with a standard deviation of 0.5 units. Across 500 such noise realizations, we observed that $\mathbb{E}[|\hat{\theta} - \theta_o|] = 0.84°$ with $\mathbf{s}_{\mathrm{opt}}$ and $15.05°$ with $\mathbf{s}_1$.

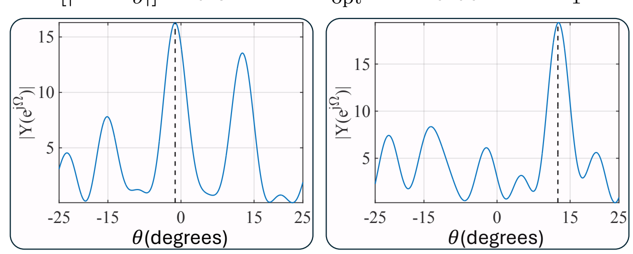



Fig. 3: Matched filtered output for $\mathbf{s}_1 = [1, 0, 0, 1, 1, 0, 0, 1]^{\mathrm{T}}$ (left) and $\mathbf{s}_{\mathrm{opt}} = [1, 1, 0, 1, 0, 0, 0, 1]^{\mathrm{T}}$ (right). We observe that the DoA $\theta_o = 15°$ can be well estimated with $\mathbf{s}_{\mathrm{opt}}$, but not using $\mathbf{s}_1$.

## References


[1] R. L. Streit, "Solution of systems of complex linear equations in the $\ell_\infty$ norm with constraints on the unknowns," *SIAM j. Sci. Stat. Comput.*, vol. 7, no. 1, pp. 132–149, 1986.