# Robust Differential Beamformers for Linear Superarrays with Non-uniformly Oriented Directional Microphones

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***Abstract*—Conventional differential beamformers for linear arrays with omnidirectional sensors cannot achieve consistent beampatterns for different steering angles. Recently, differential beamformers for linear superarrays comprising omnidirectional and directional microphones have been proposed to achieve a consistent beampattern across various steering angles. However, linear superarrays may suffer from low White Noise Gain (WNG) at low frequencies. More importantly, the WNG and, hence, the robustness of the beamformer depend on the steering angle. In this paper, we present a more general solution with non-uniformly oriented directional microphones. We employ a grid search strategy to optimize the orientations of the directional microphone array for linear superarrays, which gives the beamformer nearly identical robustness at different steering angles. Simulation results show that the proposed solution improves WNG at low frequencies (e.g., about $20$ dB improvement at $500$ Hz) without significantly affecting the directivity factor.**

***Index Terms*—Linear microphone array, Superarrays, Directional microphones**

## I. Introduction

Differential Microphone Array (DMA) processing has gained significant attention due to its advantageous characteristics, such as the ability to achieve a more frequency-invariant beampattern and a higher Directivity Factor (DF) using a small and compact array aperture [1]–[3]. DMA designs commonly utilize omnidirectional microphone elements, which can lead to a relatively low White Noise Gain (WNG) at low frequencies. Due to the low WNG, spatially white noise (such as sensor noise) is amplified in practical applications [4], [5]. To mitigate this issue, directional microphones integrated into the design of differential beamformers bring significant WNG improvement across various arrays [6]–[9].

Linear microphone arrays are a typical configuration in numerous practical applications, such as televisions, video conferencing systems, and laptops. Their widespread use has consequently drawn significant interest in linear arrays [10]–[13]. In contrast to circular arrays, differential beamformers using a linear array of omnidirectional microphones are typically only partially steerable [14]. This limitation prevents them from achieving a steering-invariant beampattern.

Recent studies [15], [16] have proposed Linear Superarrays (LSA) for differential beamformers, aiming to achieve a steerable beampattern for linear arrays. The beampattern exhibits a consistent two-dimensional shape across various steering angles and is no longer symmetrical with respect to the axis of the linear array. The LSA consists of omnidirectional and directional microphones. However, LSA continues to experience WNG challenges at low frequencies. The study outlined in [17] demonstrates that it achieves a higher WNG than LSA by incorporating a robustness constraint into the optimization problem. However, this approach significantly reduces the DF, and as a result, the beampattern is no longer frequency invariant, unlike that of LSA. Notably, the LSA's WNG is dependent on the steering angle. Especially in the best configuration utilizing both omnidirectional and bidirectional microphones [16], LSA exhibits the worst performance for WNG in the end-fire direction. The underlying reason for this can be intuitively understood: all the bidirectional microphones are oriented toward the broadside, so they inherently reject sounds originating from the end-fire direction. This rejection effectively reduces the number of "active" microphones for the end-fire sound capture, leading to a reduced WNG.

This paper presents a method to design robust differential beamformers for a broader class of Linear Superarrays, hereafter referred to as LSA+, that include directional microphones with non-uniform orientations. Optimizing the directional microphone orientations based on a grid search strategy ensures that the differential beamformer for LSA+ maintains nearly identical WNG across various steering angles. Additionally, the WNG of the beamformer based on the optimized LSA+ array is significantly higher than that of the LSA at low frequencies while obtaining a similar DF.

## II. Problem Formulation

This paper considers an LSA+ with two subarrays: the first one is a uniform linear array with $M_\text{o}$ omnidirectional microphones with inter-microphone distance $\delta_\text{o}$, and the second one is a linear array with $M_\text{d}$ directional microphones with uniform inter-microphone distance $\delta_\text{d}$ as illustrated in Fig. 1. The offset between the two subarrays is denoted as $\sigma$. Without loss of generality, we assume all microphones lie in the $x$-$y$ plane. In the following, we assume that the directional microphones in the second array have a fixed elevation angle of $\pi/2$. The

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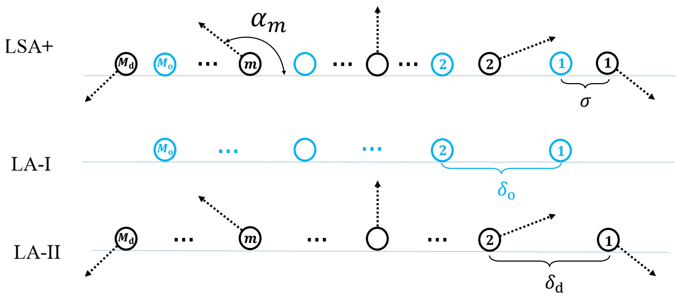



Fig. 1. An LSA+ consists of two subarrays: LA-I is a uniform linear array with $M_o$ omnidirectional microphones with inter-microphone distance $\delta_o$, and LA-II is a linear array with $M_d$ directional microphones with uniform inter-microphone distance $\delta_d$ (The arrow points in the direction of the main lobe of the directional microphone). The offset between the two subarrays is denoted as $\sigma$.

azimuth angles are represented by the vector $\boldsymbol{\alpha}$, given by:

$$\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_m, \ldots, \alpha_{M_d}]^T, \tag{1}$$

where $\alpha_m$ corresponds to the $m$-th directional microphone.

We assume a plane wave impinges on the array with an incident direction of $\Psi = (\theta,\ \phi)$ in the 3D spherical coordinate system, where $\theta$ represents the azimuth angle, and $\phi$ represents the elevation angle. The propagation vector for LSA+ is:

$$\mathbf{d}\left(\omega, \Psi, \boldsymbol{\alpha}\right) = \left[\mathbf{d}_1^T(\omega, \Psi)\ \ \mathbf{d}_2^T(\omega, \Psi, \boldsymbol{\alpha})\right]^T, \tag{2}$$

where superscript $^T$ is the transpose operator and $\mathbf{d}_1^T(\omega, \Psi)$ represents the first subarray with omnidirectional microphones, the $m$-th element for $\mathbf{d}_1(\omega, \Psi)$ is given by:

$$[\mathbf{d}_1\left(\omega, \Psi\right)]_m = e^{-j\frac{\omega}{c}((m-1)\delta_o+\sigma)\cos\theta\sin\phi}, \tag{3}$$

where $j = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, $f$ is the frequency, and $c$ is the sound speed. The propagation vector $\mathbf{d}_2(\omega, \Psi, \boldsymbol{\alpha})$ represents the second subarray with directional microphones, given by

$$\mathbf{d}_2\left(\omega, \Psi, \boldsymbol{\alpha}\right) = \left[d_1, \ldots, d_m, \ldots, d_{M_d}\right]^T. \tag{4}$$

As in [1], [18], we assume each element to have a frequency-invariant pattern defined by

$$\begin{aligned} d_m &= e^{-j\frac{\omega\delta_d}{c}(m-1)\cos\theta\sin\phi} \\ &\quad\times [p_m + (1-p_m)\cos(\theta-\alpha_m)\sin\phi], \end{aligned} \tag{5}$$

where $p_m$ defines the property of $m$-th directional microphone; for instance, it makes the well-known patterns: 1) cardioid when $p_m = 0.5$; 2) supercardioid when $p_m = 0.37$; 3) dipole when $p_m = 0$, where they are all considered as first-order directional microphones.

To obtain the symmetric sound capture ability at two end-fire directions, we assume $\alpha_m = \pi - \alpha_{M_d-m+1}$ and $p_m = p_{M_d-m+1}$ as shown in Fig. 1. For comparison, it is essential to note that the LSA assumes that all directional microphones have the same orientation, i.e., $\alpha_m = \pi/2\ \forall m \in \{1, \ldots, M_d\}$ in [15], [16]. Thus, the LSA can be considered a special case of the proposed LSA+.

The differential beamformer is categorized as a fixed beamformer, which is a time-invariant data-independent spatial filter to estimate the signal from the desired steering angles and suppress the signal from the undesired direction by applying a complex weight vector. The differential beamformer $\mathbf{h}_{\Psi_s}(\omega)$ is designed for a specific steering angle $\Psi_s = (\theta_s, \phi_s)$, where the $\theta_s$ is the target azimuth angle, and $\phi_s$ is the target elevation angle. $\mathbf{h}_{\Psi_s}(\omega)$ exhibits a distortionless response in that direction ($\Psi = \Psi_s$). In contrast, in undesired directions ($\Psi \neq \Psi_s$), the beamformer demonstrates a certain level of suppression, expressed mathematically as:

$$\mathbf{d}^H(\omega, \Psi, \boldsymbol{\alpha})\mathbf{h}_{\Psi_s}(\omega) \begin{cases} = 1, & \text{if } \Psi = \Psi_s \\ < 1, & \text{otherwise} \end{cases}, \tag{6}$$

where the superscript $^H$ is the conjugate-transpose operator.

Here, we introduce WNG, which can quantify the robustness of differential beamformers [3], expressed as:

$$\mathcal{W}[\mathbf{h}_{\Psi_s}(\omega)] = \frac{\left|\mathbf{d}^H(\omega, \Psi_s, \boldsymbol{\alpha})\mathbf{h}_{\Psi_s}(\omega)\right|^2}{\mathbf{h}_{\Psi_s}^H(\omega)\mathbf{h}_{\Psi_s}(\omega)}. \tag{7}$$

This paper considers designing differential beamformers with various steering angles $\theta$. For brevity, we will use $\mathbf{h}_{\theta_s}(\omega)$ to denote the $\mathbf{h}_{\Psi_s}(\omega)$ with $\Psi_s = (\theta_s, \phi_s = \pi/2)$. When designing a set of $Q$ differential beamformers $\mathbf{h}_{\theta^1}(\omega), \ldots, \mathbf{h}_{\theta^Q}(\omega)$ for various azimuth angles $\theta_s^1, \ldots, \theta_s^Q$, the problem using LSA is that the WNG for each differential beamformer differs, i.e.,

$$\mathcal{W}[\mathbf{h}_{\theta^1}(\omega)] \neq \cdots \neq \mathcal{W}[\mathbf{h}_{\theta^Q}(\omega)]. \tag{8}$$

This paper focuses on designing differential beamformers with consistent WNGs for different steering angles $\theta_s^1, \ldots, \theta_s^Q$ using the proposed LSA+.

## III. Proposed Method

We leverage a null-constrained method [2], [19] to design the differential beamformers with the proposed LSA+. We formulate the problem as a linear system of equations as below:

$$\mathbf{R}(\omega, \boldsymbol{\alpha})\mathbf{h}_{\theta_s}(\omega) = \mathbf{c}_{\theta_s}, \tag{9}$$

where $\mathbf{h}_{\theta_s}(\omega)$ is the LSA+ beamforming weights we want to obtain, and $\mathbf{c}_{\theta_s}$ is a vector containing $N$ null constraints of the differential beamformer, defined as:

$$\mathbf{c}_{\theta_s} = [c_{\theta_1}, \ldots c_{\theta_n}, \ldots, c_{\theta_N}]^T, \tag{10}$$

where $c_{\theta_n}$ is usually defined as:

$$c_{\theta_n} = \begin{cases} 1, & \text{if } \theta_n = \theta_s \\ 0, & \text{otherwise} \end{cases}, \tag{11}$$

where $\theta_n \neq \theta_s$ decides the null positions of the beampattern and $\theta_n = \theta_s$ defines the steering angle.

The constraint matrix $\mathbf{R}(\omega, \boldsymbol{\alpha})$ of size $N \times (M_d + M_o)$ is given by

$$\mathbf{R}(\omega, \boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{d}^H(\omega, \Psi_1, \boldsymbol{\alpha}) \\ \vdots \\ \mathbf{d}^H(\omega, \Psi_N, \boldsymbol{\alpha}) \end{bmatrix}, \tag{12}$$

**Algorithm 1** Grid search strategy to optimize the orientations $\boldsymbol{\alpha}$ for the LSA+

**Input:** One angular frequency $\omega$ and $p_1 \cdots p_{M_\mathrm{d}}$.
**Output:** The optimal orientations $\boldsymbol{\alpha_o}$
**for** $\alpha_1 = 0$ *to* $2\pi$ **do**
  ⋮
  **for** $\alpha_m = 0$ *to* $2\pi$ **do**
    ⋮
    **for** $\alpha_{\left\lfloor \frac{M_d}{2} \right\rfloor} = 0$ *to* $2\pi$ **do**
      Define $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_m, \ldots, \alpha_{M_\mathrm{d}}]^T$
      **for** *each steering angle* $\theta_s$ **do**
        Calculate the beamformer $\mathbf{h}_{\theta_\mathrm{s}}(\omega)$ using (13)
        Calculate the WNG $\mathcal{W}[\mathbf{h}_{\theta_\mathrm{s}}(\omega)]$ using (7)
      Construct $\boldsymbol{v}$ with the WNGs for all angles $\theta_\mathrm{s}$
      Obtain $I(\boldsymbol{\alpha}) = \text{standard deviation}(\boldsymbol{v})$
Find the final $\boldsymbol{\alpha_o}$ that minimizes $I(\boldsymbol{\alpha})$

where $\mathbf{d}^H(\omega, \Psi_n, \boldsymbol{\alpha}), n = 1, 2, \ldots, N$, is the propagation vector of length $M_\mathrm{d} + M_\mathrm{o}$ defined in (2). Here, we define $\Psi_n = (\theta_n, \phi_n)$ with a fixed $\phi_n = \pi/2$.

To achieve the maximum WNG for robust beamforming, we take the well-known minimum-norm solution in [2], [3] to solve our linear system equations as shown in (9). Such that the proposed LSA+ differential beamformer is obtained by:

$$\mathbf{h}_{\theta_\mathrm{s}}(\omega) = \mathbf{R}^H(\omega, \boldsymbol{\alpha})[\mathbf{R}(\omega, \boldsymbol{\alpha})\mathbf{R}^H(\omega, \boldsymbol{\alpha})]^{-1}\mathbf{c}_{\theta_\mathrm{s}}. \tag{13}$$

Next, we propose a grid search method to optimize the orientations $\boldsymbol{\alpha}$, ensuring that beamformers designed for different steering angles $\theta$ have nearly identical WNGs. For a specific frequency bin $\omega$ and a set of directional microphones $p_1, \ldots, p_{M_\mathrm{d}}$ in the LSA+, we perform a grid search for each steering angle $\alpha_m$ in the range from 0 to $2\pi$ in a predefined step such as $\pi/180$. Given our assumption that $\alpha_m = \pi - \alpha_{M_\mathrm{d}-m+1}$ for a symmetric design, we only perform the grid search for $\alpha_m$ with $m$ ranging from 1 to $\left\lfloor \frac{M_\mathrm{d}}{2} \right\rfloor$.

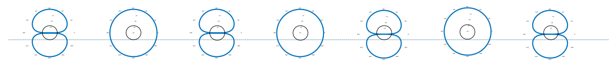
For a specific configuration $\boldsymbol{\alpha}$ during the search, we calculate the beamformers $\mathbf{h}_{\theta_\mathrm{s}}(\omega)$ using equation (13) for each steering angle $\theta_\mathrm{s}$. Subsequently, we compute the corresponding WNG $\mathcal{W}[\mathbf{h}_{\theta_\mathrm{s}}(\omega)]$ using equation (7). For a set of steering angles $\theta_\mathrm{s}$, we obtain a vector $\boldsymbol{v}$ that contains all WNGs. We then calculate the standard deviation of this vector $\boldsymbol{v}$, which is denoted by $I(\boldsymbol{\alpha})$.

After completion of the grid search, we obtain the standard deviations for all possible configurations of $\boldsymbol{\alpha}$ and select the final $\boldsymbol{\alpha_o}$ that minimizes the standard deviation across all values. **Algorithm 1** summarizes this grid search strategy.

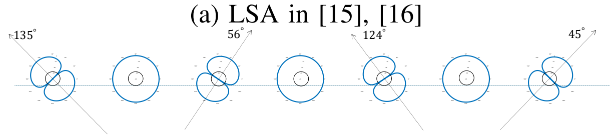
## IV. Performance Evaluation

In this section, we study the proposed LSA+'s performance in terms of beampattern, WNG, and DF. First, we define the beampattern and DF used in the evaluation.

Beampattern illustrates the directional sensitivity of a beamformer to a plane wave impinging on the array from a direction



(a) LSA in [15], [16]




(b) An optimized example of the proposed LSA+

Fig. 2. Array setup comparison

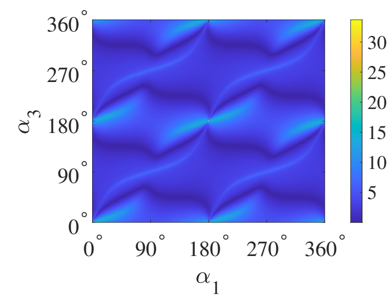
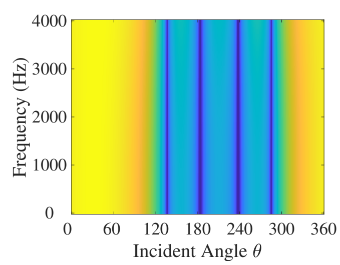
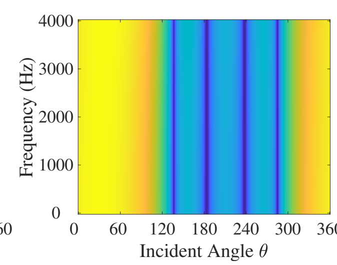



Fig. 3. Heatmap showing the $I(\boldsymbol{\alpha})$ regarding the set-up $\alpha_1$ and $\alpha_3$.




(a) Proposed LSA+




(b) Conventional LSA

Fig. 4. Beampatterns of the proposed LSA+ and conventional LSA for a steering angle of 30°. The proposed LSA+ maintains frequency invariant beampatterns as the conventional LSA.

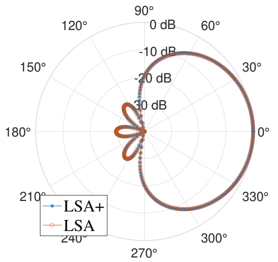
in the 3D spherical coordinate system [20]. $\mathcal{B}[\mathbf{h}_{\Psi_\mathrm{s}}(\omega), \Psi]$ denotes the beampattern defined as:

$$\mathcal{B}(\mathbf{h}_{\theta_\mathrm{s}}(\omega), \Psi) = \mathbf{d}^H(\omega, \Psi, \boldsymbol{\alpha})\, \mathbf{h}_{\theta_\mathrm{s}}(\omega). \tag{14}$$

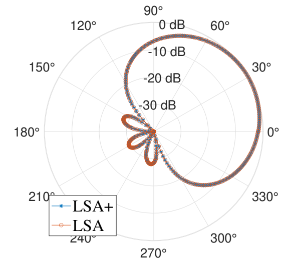
The DF is defined as the ratio between the output signal power in the desired steering angle and the power averaged over all directions [20]:

$$\mathcal{DF}[\mathbf{h}_{\theta_\mathrm{s}}(\omega)] = \frac{|\mathcal{B}[\mathbf{h}_{\theta_\mathrm{s}}(\omega), \Psi_\mathrm{s}]|^2}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} |\mathcal{B}[\mathbf{h}_{\theta_\mathrm{s}}(\omega), \Psi]|^2 \cdot \sin\phi \, d\phi \, d\theta}. \tag{15}$$

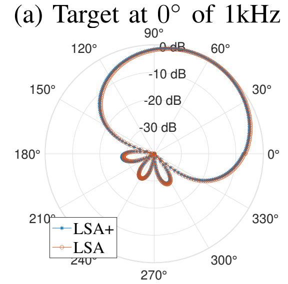
For comparison, we select the optimal LSA configuration from [15], [16]. The LSA setup example, shown in Fig. 2(a), features three omnidirectional and four bidirectional microphones. The directional microphones in the LSA all have an orientation of $\pi/2$. Both LSA and LSA+ exhibit a uniform inter-element spacing of 0.5 cm and share the same second-order differential beamformer design, characterized by specified null positions at $[\theta_\mathrm{s} - \frac{106}{180}\pi, \theta_\mathrm{s} + \frac{106}{180}\pi, \theta_\mathrm{s} - \frac{153}{180}\pi, \theta_\mathrm{s} + \frac{153}{180}\pi]$ for $\mathbf{h}_{\theta_\mathrm{s}}(\omega)$ (the same design example in [16]). Lastly, we



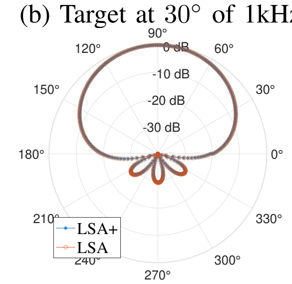

(a) Target at 0° of 1kHz




(b) Target at 30° of 1kHz




(c) Target at 60° of 1kHz




(d) Target at 90° of 1kHz

Fig. 5. Steerability on beampatterns of the proposed LSA+ and conventional LSA. The proposed LSA+ maintains steerability similar to that of a conventional LSA. The beampattern is no longer symmetrical with respect to the axis of the linear array and exhibits a consistent shape across various steering angles.

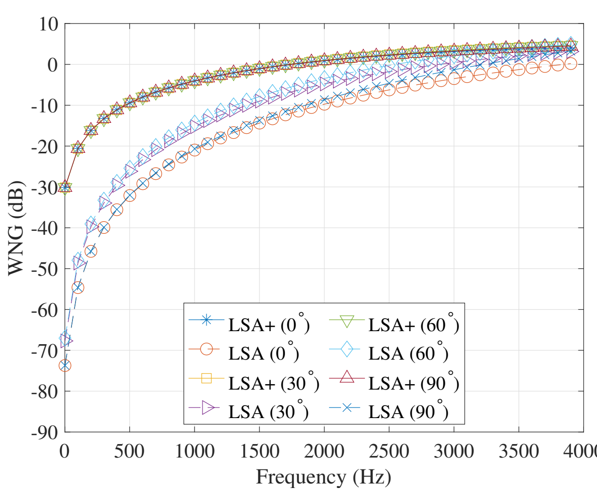



Fig. 6. Comparisons of WNG between the proposed LSA+ and conventional LSA.

choose the same number and type of microphone as LSA to design the proposed LSA+ for comparison.

So far, we aim to optimize the setup $\boldsymbol{\alpha} = [\alpha_1, 0, \alpha_3, 0,\ \pi - \alpha_3, 0, \pi - \alpha_1]^T$ for the orientations of the bidirectional microphones in the LSA+. In this paper, we present a design example where we apply the grid search strategy at 1 kHz to minimize the variance of the white noise gains for beamformers targeted at different steering angles, specifically $\theta_s \in \{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\}$. This approach allowed us to obtain a global $I(\boldsymbol{\alpha})$ regarding $\boldsymbol{\alpha}$ shown in Fig. 3. To minimize the $I(\boldsymbol{\alpha})$, the optimized orientation vector $\boldsymbol{\alpha_o}$ follows

$$\boldsymbol{\alpha_o} = \left[\frac{135}{180}\pi, 0, \frac{56}{180}\pi, 0, \pi - \frac{56}{180}\pi, \pi - \frac{135}{180}\pi\right], \qquad (16)$$

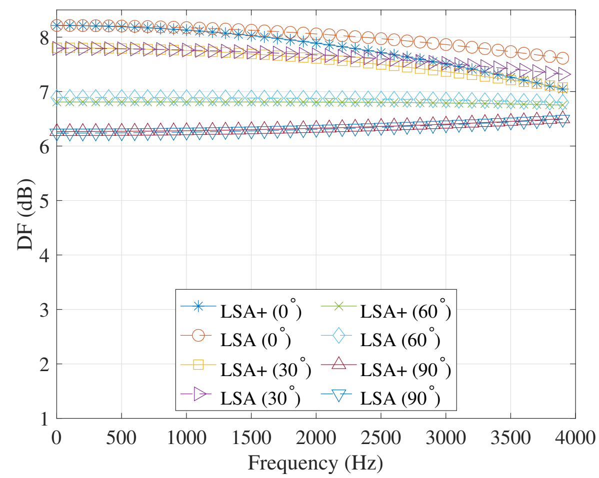



Fig. 7. Comparisons of DF between the proposed LSA+ and conventional LSA.

which corresponds to the angular degrees [135°, 0, 56°, 0, 124°, 0, 45° ] in Fig. 2(b).

In Fig. 4, we study the beampattern in 2D via setting $\phi = \frac{\pi}{2}$ for $\Psi$ in (14) as widely done in DMA area [1], [2], [15], [16]. We show the beampatterns between the proposed LSA+ and the conventional LSA at a steering angle of 30°, where the LSA+ retains the frequency-invariant property as the LSA. In Fig. 5, the desired beamforming steering angle in (a, b, c, d) is set to $\theta_s = 0°, 30°, 60°$ and $90°$ respectively. Both designs have nearly identical steerability at 1 kHz. Since the proposed LSA+ exhibits frequency invariance as the LSA, we can conclude that the LSA+ has similar steerability of the beampattern as the LSA.

Figures 6 and 7 compare the proposed LSA+ with the conventional LSA in terms of WNG and DF for various beamforming steering angles at $\theta_s = 0°$, $30°$, $60°$, and $90°$. From Fig. 6, we observe that the proposed LSA+ maintains nearly identical WNG across different steering angles, while the conventional LSA shows varying WNG levels. Notably, LSA exhibits the lowest WNG at the end-fire beamforming direction. Additionally, the WNG of LSA+ is significantly higher at low frequencies compared to all WNG levels of the conventional LSA, about $+20$dB at 500Hz. Furthermore, Fig. 7 demonstrates that LSA+ achieves a similar level of directivity as the conventional LSA. However, both LSA+ and conventional LSA exhibit different DF levels across various steering angles. Since we calculate the DFs in 3D via (15), beampatterns of LSA and LSA+ in 3D at different steering angles may not have identical shapes. As shown in Fig. 5, the beampatterns are steering angle-independent for both the LSA and LSA+ when considering an elevation of $\phi = \pi/2$. However, as indicated by the DF, the 3D beampattern is steering angle-dependent. In conclusion, the proposed LSA+ achieves significantly higher and more uniform WNG than the conventional LSA while providing comparable DF performance.

## V. Conclusions

This paper presents a method for designing a robust differential beamformer for LSA with non-uniformly oriented directional microphones, i.e., LSA+. Through a grid search strategy, we can find an optimum microphone orientation in terms of the WNG variation across different steering angles. Unlike LSA, our optimized LSA+ design ensures that the beamformers for different steering angles maintain nearly identical WNG. Additionally, the WNG of the LSA+ is significantly higher compared to the varying WNGs of the conventional LSA. Overall, our method provides improved and consistent robustness across different steering angles while obtaining a similar directivity factor.

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