GSN-FxlogLMS+: A Step-Size Normalized Algorithm for Robust Impulsive Noise Control

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***Abstract*—Active Noise Control (ANC) under impulsive noise conditions remains a significant challenge. The Filtered-x logLMS (FxlogLMS) algorithm suppresses large noise spikes through logarithmic transformation, but suffers from slow convergence and limited robustness in highly impulsive environments.**

This paper proposes GSN-FxlogLMS+, a novel algorithm that integrates a variable step-size scheme with a modified robust error function. The proposed method dynamically adapts to noise conditions without requiring manually set thresholds, achieving a better balance between robustness and convergence speed.

Simulation results demonstrate that GSN-FxlogLMS+ improves noise suppression performance by up to 2.8 dB, compared to conventional methods, particularly under highly impulsive noise conditions.

I. Introduction

Active Noise Control (ANC) is widely applied in industrial noise reduction, personal hearing protection, and automotive systems. The Filtered-x Least Mean Square (FxLMS) algorithm is a standard approach due to its computational efficiency and adaptability to Gaussian noise. However, its performance significantly degrades under impulsive noise, which exhibits heavy-tailed, non-Gaussian characteristics.

To address this, various modifications have been explored. Thresholding-based methods [1], [2] and normalized step-size techniques [3], [4] suppress extreme noise amplitudes, improving stability at the cost of slower convergence. Robust Error Function (REF)-based approaches enhance robustness by applying nonlinear mappings to compress large error amplitudes [5]–[7]. While these methods mitigate moderate impulsive noise, their suppression saturates at extreme outliers, limiting adaptability. The FxgsnLMS algorithm [8] models impulsive noise using a generalized Gaussian distribution with adaptive step-size normalization, improving stability under α-stable noise conditions.

Despite these advancements, existing methods struggle to balance robustness, convergence speed, and computational efficiency. To overcome this, we propose GSN-FxlogLMS+, which integrates an adaptive robust error function based on FxlogLMS [9] with step-size normalization. The proposed method mitigates impulsive noise while maintaining fast adaptation, without requiring predefined thresholds or additional constraints. Experimental results confirm that GSN-FxlogLMS+ outperforms conventional methods in both robustness and convergence speed, particularly in highly impulsive environments.

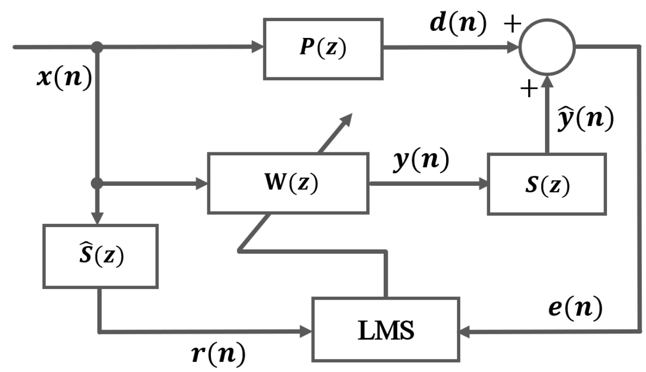


Fig. 1: Block diagram of a feedforward ANC system based on the FxLMS algorithm.

II. Conventional Feedforward Active Noise Control

A. Feedforward Active Noise Control System

Figure 1 illustrates the structure of a feedforward ANC system. In this system, the reference microphone and error microphone capture the reference signal $x(n)$ and the error signal $e(n)$, respectively. The secondary loudspeaker generates the control signal $y(n)$ to attenuate the primary noise $d(n)$. The transfer functions $P(z)$ and $S(z)$ represent the primary and secondary paths, respectively. The secondary path model $\hat{S}(z)$ is pre-estimated, and $W(z)$ denotes the noise control filter.

The filtered reference signal $r(n)$, which passes through the secondary path model, is given by

$$r(n) = \sum_{j=0}^{L-1} \hat{s}(j)x(n-j), \tag{1}$$

where $\hat{s}(n)$ is the impulse response of the secondary path with a length of L.

The k-th coefficient of the noise control filter in the FxLMS algorithm, $w_k(n)$, is updated as

$$w_k(n+1) = w_k(n) - \mu e(n)r(n), \tag{2}$$

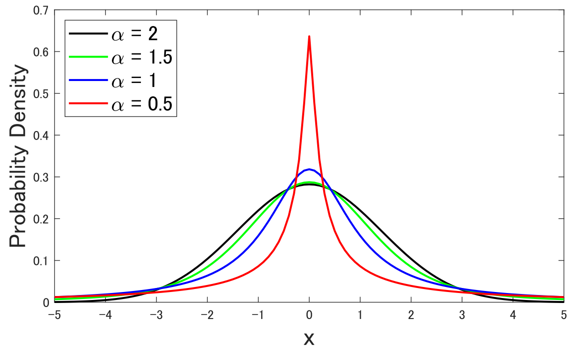


Fig. 2: Probability density functions of the SαS distribution for different values of α: $\alpha = 0.5, 1, 1.5, 2$.

where μ is the step-size parameter.

B. α-Stable Noise

Impulsive noise is commonly generated by press machines, electric hammers, and various industrial machinery. The health effects of such noise are significant. The aim of this paper is to develop an effective coefficient update algorithm for mitigating impulsive noise.

Impulsive noise is often modeled using a symmetric α-stable (SαS) distribution, defined as

$$\psi(t) = \mathrm{e}^{-\gamma|t|^{\alpha}}, \tag{3}$$

where $0 < \alpha < 2$ is the characteristic exponent controlling the kurtosis of the Probability Density Function (PDF), and $\gamma > 0$ is the scale factor. In this paper, γ is set to 1.

As α approaches 0, the distribution exhibits strong impulsiveness, whereas for $\alpha \to 2$, it converges to a Gaussian distribution. Some examples of SαS distributions are shown in Fig. 2.

III. Modified FxlogLMS Algorithm for Active Impulsive Noise Control

Various methods have been proposed to address the limitations of the FxLMS algorithm. This section reviews conventional ANC approaches designed for impulsive noise.

A. Clipping

Clipping-based algorithms limit the reference and error signals within predefined thresholds. The Th-FxLMS algorithm [1] updates the filter coefficients as

$$w_k(n+1) = w_k(n) - \mu\,\mathcal{C}(e(n)) \sum_{j=0}^{L-1} \hat{s}(j)\,\mathcal{C}(x(n)). \tag{4}$$

Here, the clipping function is defined as

$$\mathcal{C}(x) = \max(c_2, \min(c_1, x)), \tag{5}$$

where $c_1, c_2 > 0$ are the upper and lower thresholds.

By applying $\mathcal{C}(\cdot)$, extreme values in $x(n)$ and $e(n)$ are suppressed, preventing instability in the filter update. However, fixed thresholds require dynamic adjustment for optimal performance.

B. Variable Step-Size

A large step-size accelerates convergence but increases steady-state misadjustment. The Variable Step-Size (VSS) approach addresses this trade-off by dynamically adjusting the step-size.

One widely used VSS method is the Normalized FxLMS (FxNLMS) algorithm [3], which adjusts the step-size μ in Eq. (2) based on the squared norm of the filtered reference signal:

$$\hat{\mu}(n) = \frac{\mu}{\|\mathbf{r}(n)\|_2^2 + \delta}, \tag{6}$$

where δ is a small positive constant to prevent instability due to division by zero.

To further enhance stability, the FxLMS algorithm with Normalized Step-Size (NSS) [3] incorporates the estimated mean power of the error signal $E_e(n)$:

$$\hat{\mu}(n) = \frac{\mu}{\|\mathbf{r}(n)\|_2^2 + E_e(n) + \delta}, \tag{7}$$

where $E_e(n)$ is estimated using an exponential moving average:

$$E_e(n) = \lambda E_e(n-1) + (1-\lambda)e^2(n), \tag{8}$$

with forgetting factor λ $(0 < \lambda < 1)$.

The General Step-size Normalized FxLMS (FxgsnLMS) algorithm [8] introduces a Gaussian-based normalization to balance stability and convergence speed. The step-size is dynamically adapted as

$$\hat{\mu}(n) = \frac{\mu}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2(n)}{2\sigma^2}\right), \tag{9}$$

where σ is the standard deviation. As shown in Eq. (9), the step-size decreases exponentially with $x^2(n)$, limiting adaptation under extreme noise conditions.

However, VSS methods rely on statistical properties of the input signal power, making them less effective against sudden, high-amplitude impulsive noise. Since impulsive noise deviates significantly from typical signal statistics, these methods fail to adjust the step-size appropriately, resulting in insufficient robustness in highly non-Gaussian environments.

C. Robust Error Function

The Robust Error Function (REF) introduces a non-linear mapping to suppress large error amplitudes, enhancing robustness against impulsive noise.

One approach minimizes a squared logarithmic cost function, leading to the update equation:

$$w_k(n+1) = w_k(n) - \mu\,\mathrm{sgn}(e(n)) \frac{\log|\hat{e}(n)|}{|\hat{e}(n)|} r(n), \tag{10}$$

where $\hat{e}(n) = \max(1, |e(n)|)$. This approach prevents excessive adaptation but stops updating when $|e(n)| < 1$, which may slow convergence under mild noise conditions.

Another approach employs a hyperbolic cosine cost function, as in the FxLCH algorithm [7], which is given by:

$$w_k(n+1) = w_k(n) - \hat{\mu}(n) \tanh(\rho e(n)) r(n), \tag{11}$$

where $\hat{\mu}(n)$ follows Eq. (6). This method smooths the suppression of large errors using the $\tanh$ function, but its output saturates for extreme values, limiting adaptation to sudden noise variations.

While REF-based methods improve robustness against impulsive noise, they inherently reduce sensitivity to all errors, slowing convergence and degrading tracking performance in dynamic noise environments.

IV. Proposed Method

As discussed in Sec. 3, both Variable Step-Size (VSS) and Robust Error Function (REF) methods exhibit a significant trade-off between robustness against impulsive noise and convergence speed.

To overcome this trade-off, we develop a new REF-based algorithm, FxlogLMS+, which improves convergence speed while maintaining the robustness of FxlogLMS. Furthermore, to further enhance convergence, we introduce an optimal VSS approach for FxlogLMS+, utilizing the step-size normalization technique of the FxgsnLMS algorithm. By integrating these techniques, we propose the General Step-size Normalized FxlogLMS+ (GSN-FxlogLMS+) algorithm, which effectively balances noise robustness and fast adaptation.

The update equation of the GSN-FxlogLMS+ is given by

$$w_k(n+1) = w_k(n) - \hat{\mu}(n)\frac{\psi(e(n))r(n)}{\sqrt{\mathrm{E}[x(n)^2]}} \quad (12)$$

$$\psi(e(n)) = \mathrm{sgn}(e(n))\frac{\log(G|e(n)|+1)}{G|e(n)|+1}, \quad (13)$$

where G is the gain factor, which enhances convergence speed at the cost of increased misadjustment after convergence, and $\hat{\mu}(n)$ follows Eq. (9). The Root Mean Square (RMS) value of the reference signal $x(n)$ is introduced for normalization to enhance the algorithm's robustness against impulsive noise by scaling down the update magnitude during the occurrence of high-amplitude impulses.

Unlike FxlogLMS and Th-FxLMS, the proposed algorithm mitigates sensitivity to impulsive noise without requiring predefined thresholds or explicit constraints on $e(n)$. By smoothly shaping the robust error function, the method adapts dynamically to various noise conditions, offering improved stability and flexibility.

Let $J(n)$ denote the cost function and $\phi(e) = \partial J(e)/\partial e$ represent the score function, which corresponds to the robust error function. Figure 3 illustrates the relationship between the error signal and the robust error functions $\phi(e)$ for both the proposed and conventional methods. The proposed method effectively suppresses large errors, reducing excessive adaptation under impulsive noise, while retaining the behavior of standard FxLMS for small errors. The gain factor G allows tuning of this balance.

A. Selection of the Variable Step-size method

To validate the effectiveness of GSN as a variable step-size method for FxlogLMS+, we compare its performance with

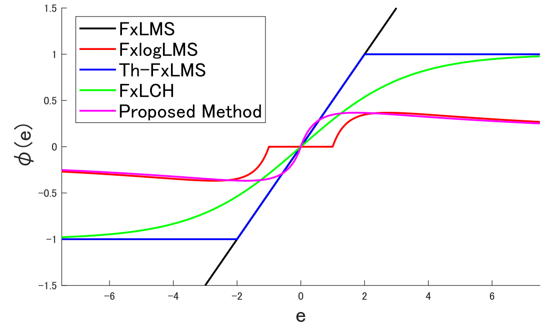


Fig. 3: Comparison of score functions of the proposed and conventional methods.

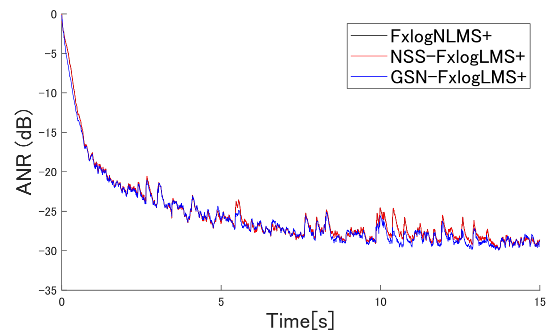


Fig. 4: Comparison of step-size normalization methods in FxlogLMS+.

two conventional methods: NLMS-type and NSS-type method. Each method is integrated into FxlogLMS+. .

Figure 4 shows the time behavior of the averaged noise reduction (ANR) under impulsive noise conditions using α-stable noise with $\alpha = 1.85$ (as shown in Fig. 6). ANR is defined as

$$\mathrm{ANR}(n) = 20\log\frac{A_e(n)}{A_d(n)}, \quad (14)$$

where

$$A_e(n) = \chi A_e(n-1) + (1-\chi)|e(n)|, \quad (15)$$

$$A_d(n) = \chi A_d(n-1) + (1-\chi)|d(n)|, \quad (16)$$

with $\chi = 0.999$.

The NLMS-type (black line) and NSS-type (red line) normalizations exhibit similar performance, with their ANRs almost identical and overlapping, providing fast initial convergence but lacking robustness against impulsive noise. In contrast, the GSN-based approach (blue line) achieves both high robustness and fast adaptation, making it the most effective step-size method for FxlogLMS+.

Based on this result, we employ the GSN-based VSS in the proposed GSN-FxlogLMS+ algorithm.

B. Optimization of the Gain Factor G

The gain factor G controls the trade-off between convergence speed and robustness against impulsive noise. Smaller

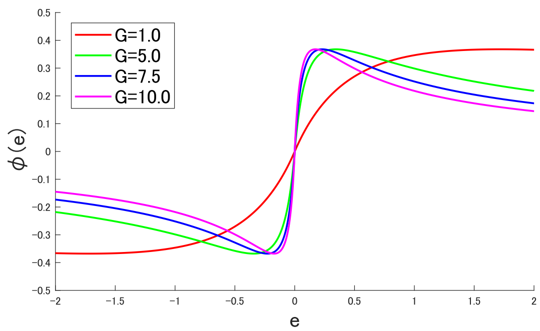


Fig. 5: Effect of G on the score function.

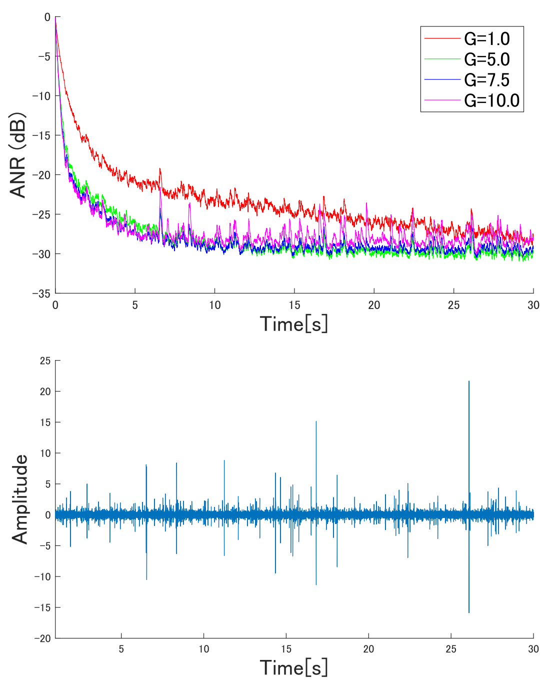


Fig. 6: ANR behavior for different G values.

values improve robustness but slow adaptation, while larger values accelerate convergence at the cost of stability.

To determine the optimal G, we first analyzed its effect on the score function, which represents the derivative of the cost function and governs the update magnitude of the adaptive filter. Figure 5 illustrates how different G values affect the score function. As G increases, adaptation to small errors accelerates, improving convergence speed and robustness. However, excessive convergence speed may lead to increased misadjustment.

To further evaluate the impact of G, we examined the ANR behavior under α-stable noise ($\alpha = 1.85$), as shown in Figure 6.

The results indicate that $G = 7.5$ achieves a favorable balance between convergence speed and robustness. Thus, we adopt $G = 7.5$ in the subsequent experiments.

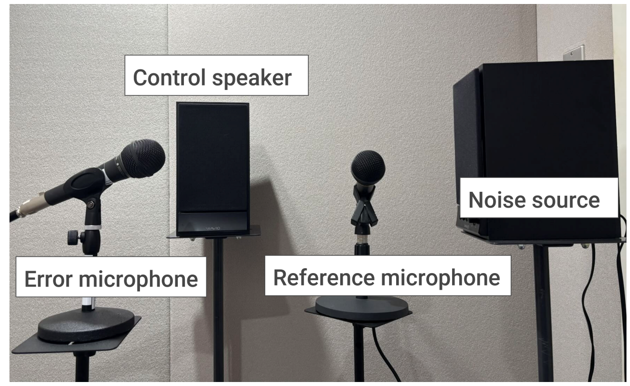


Fig. 7: Experiment Environment.

V. EXPERIMENT

To evaluate the effectiveness of the proposed method, we conducted simulation experiments using α-stable noise. The primary noise duration was set to 30 seconds at a 16 kHz sampling rate. The primary and secondary path lengths were 512 and 128 samples, respectively, with corresponding physical lengths of 20 cm and 5 cm. The secondary path was assumed to be perfectly estimated, i.e., $\hat{S}(z) = S(z)$). Figure 7 illustrates the experimental setup. The noise control filter length was 512 samples.

A. *Performance Comparison*

We evaluated the proposed method under various α-stable noise conditions: $\alpha = 1.65$ (Case 1), $\alpha = 1.45$ (Case 2), and time-varying α (Case 3). Table I lists the compared methods and parameter settings.

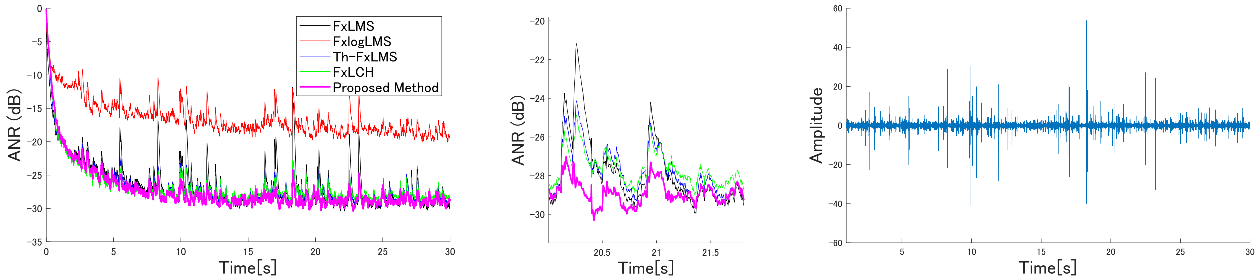
TABLE I: Parameter settings of algorithms.

Algorithm	**Parameters**
FxLMS	$\mu = 0.3$
FxlogLMS	$\mu = 0.5$
Th-FxLMS	$\mu = 0.4$, $c_1, c_2 = 0.1$
FxLCH	$\mu = 0.03$, $\rho = 10$
GSN-FxlogLMS+	$\mu = 0.016$, $G = 7.5$, $\lambda = 0.99$

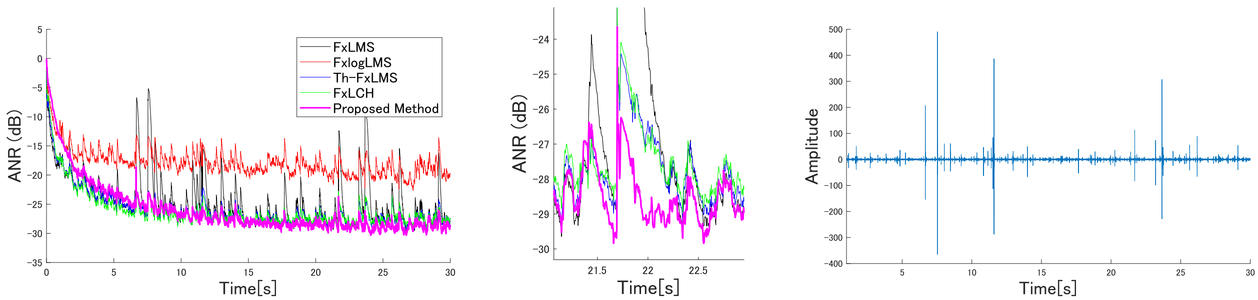
Figures 8 show the ANR results for each case. As the noise becomes more impulsive, higher robustness is required. In Case 3, where α varies over time, algorithms must adapt dynamically without reinitialization, making it particularly challenging.

The results indicate that the proposed method effectively suppresses noise, even in high-kurtosis cases such as $\alpha = 1.45$. Compared to the best-performing conventional method, GSN-FxlogLMS+ achieved an additional 2.8 dB improvement in ANR under highly impulsive conditions (Case 2). This demonstrates its superior capability to handle extreme noise spikes, while maintaining stable adaptation throughout the entire signal duration.

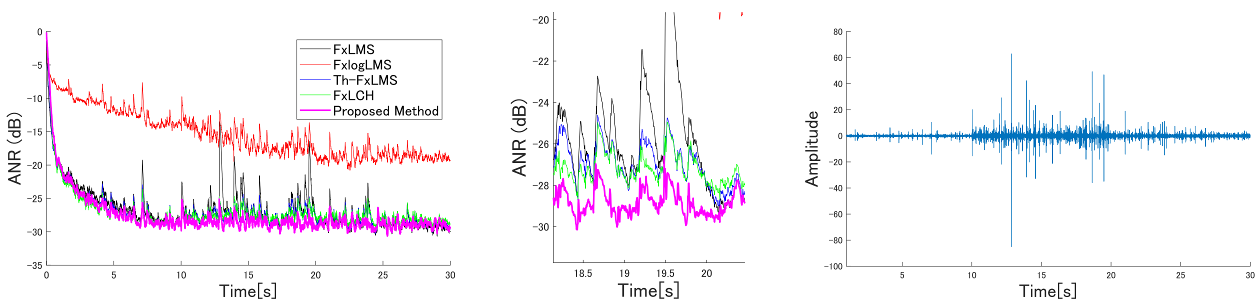
Particularly in Case 3, where α varies over time, GSN-FxlogLMS+ outperformed all conventional methods, confirming its ability to adapt dynamically without re-initialization.



(a) Case 1 : $\alpha = 1.65$ (left: ANR, center: enlarged ANR, right: waveform of primary noise).



(b) Case 2 : $\alpha = 1.45$ (left: ANR, center: enlarged ANR, right: waveform of primary noise).



(c) Case 3 : $\alpha = 1.85, 1.45, 1.65$ (left: ANR, center: enlarged ANR, right: waveform of primary noise).

Fig. 8: ANR results for each case.

VI. Conclusion

Despite these advancements, existing methods still struggle to balance robustness, convergence speed, and computational efficiency.

To address this, we propose GSN-FxlogLMS+, which combines an adaptive robust error function with step-size normalization. Our method enhances stability under impulsive noise while maintaining fast adaptation, without requiring manually set thresholds or additional constraints.

Experimental evaluations demonstrate that GSN-FxlogLMS+ surpasses conventional methods in terms of both robustness and convergence speed, particularly in highly impulsive environments.

References

[1] M.T. Akhtar and W. Mitsuhashi, "Improved adaptive algorithm for active noise control of impulsive noise," 9th International Conference on Signal Processing, pp. 2669–2672, 2008.

[2] G. Sun, M. Li, and T. C. Lim, "Enhanced filtered-x least mean M-estimate algorithm for active impulsive noise control," Appl. Acoust., vol. 97, pp. 96-103, 2015.

[3] M.T. Akhtar and W. Mitsuhashi, "A modified normalized FxLMS algorithm for active control of impulsive noise," European Signal Processing Conference (EUSIPCO), pp. 1-5, 2010.

[4] L. Wang, K. Chen, and J. Xu, "Convex combination of the FxAPV algorithm for active impulsive noise control," Appl. Acoust., vol. 195, Art. no. 108665, 2022.

[5] X. Zhang, Y. Wang, and Z. Liu, "Logarithmic hyperbolic cosine adaptive filter and its performance analysis," IEEE Trans. Signal Process., vol. 70, pp. 1234-1245, 2023.

[6] O. M. Abdelrhman and L. Sen, "Robust adaptive filtering algorithms based on the half-quadratic criterion," *Signal Process.*, vol. 204, Art. no. 108148, 2023.

[7] A. Mirza, A. Zeb, M. Y. Umair, D. Iyas, and S.A. Sheikh, "Less complex solutions for active noise control of impulsive noise," Anal. Integr. Circuits Signal Process., vol. 102, no. 3, pp. 507-521, 2020.

[8] Y. Zhou, Q. Zhang, and Y. Yin, "Active control of impulsive noise with symmetric α-stable distribution based on an improved step-size normalized adaptive algorithm," Mechanical Systems and Signal Processing, vol. 56, pp. 320-339, 2015

[9] L. Wu, H. He, and X. Qiu, "An active impulsive noise control algorithm with logarithmic transformation," IEEE Trans. Audio Speech Lang. Process., vol. 19, no. 4, pp. 1041–1044, 2011.