Super-Resolution of Depth Maps Using Graph Regularization

George Gartzonikas
Department of CSE
University of Ioannina
Ioannina, Greece
0009-0001-0972-4131

Evaggelia Tsiligianni
Department of CSE
University of Ioannina
Ioannina, Greece
0000-0003-2876-5788

Nikos Deligiannis
Electronics and Informatics Dept.
Vrije Universiteit Brussel & IMEC
Brussels, Belgium
0000-0001-9300-5860

Lisimachos P. Kondi
Department of CSE
University of Ioannina
Ioannina, Greece
0000-0002-0678-4526

***Abstract*—We address single depth map super-resolution as an inverse problem and rely on a graph-based representation of depth maps to explore the use of the Laplacian matrix as a regularizer. The Laplacian matrix is well known in graph theory for encoding important properties of graph nodes and edges. We solve the corresponding optimization problem with the Alternating Direction Method of Multipliers (ADMM), an algorithm that has been widely used in recent years for solving complex problems by splitting them into smaller and simpler subproblems. By using the graph Laplacian as a regularizer within the ADMM algorithm, we promote smoothness and preserve structural details of the considered depth map. We showcase a comprehensive formulation of the ADMM algorithm and how the Laplacian matrix is integrated. Results show that our approach outperforms existing optimization-based solutions and ADMM-based methods that use machine learning techniques for regularization (Plug and Play priors).**

***Index Terms*—depth maps, super-resolution, ADMM, Laplacian matrix**

I. Introduction

Depth map super-resolution (SR) aims to reconstruct high-resolution (HR) depth maps from their low-resolution (LR) counterparts. Depth maps play a crucial role in various applications, such as autonomous driving [1] and 3D reconstruction [2]. However, due to hardware limitations and sensor noise, captured depth maps often suffer from low resolution and artifacts, posing serious issues for various 3D applications. In general, depth cameras are equipped with an additional RGB sensor that can capture color images with a resolution higher than that of depth maps. Most of the existing works apply guided depth map SR, that is, they employ the HR color counterpart as guidance for the SR task. Nevertheless, in low-light conditions the guidance image is usually noisy and may mislead the depth restoration algorithm, making the color assisted approaches less general [3], [4]. Over the years, many approaches have been proposed to enhance depth map resolution, ranging from classical interpolation methods to deep learning-based techniques [5]. Nevertheless, achieving an optimal balance between computational efficiency and fidelity remains an open challenge.

Different from color images, depth maps do not contain rich texture information, but they are piecewise smooth, that is, they contain smooth areas separated by sharp boundaries. Unlike natural image SR, where perceptual quality is a primary concern, depth map SR must preserve geometric accuracy and structural consistency to ensure reliable 3D information.

Similar to color image SR, analytical approaches for depth map SR address the estimation of the HR depth map from the observed LR counterpart as an inverse problem and introduce various priors [5]. The corresponding optimization methods apply different regularization techniques. Total variation (TV) regularization aims to minimize the gradient magnitude to enforce smoothness. Although the TV constraint helps to distinguish the edge and the noise, it tends to generate the "staircasing artifacts" for the smooth region [5]. Recent studies have shown the powerful capability of the graph Laplacian to deal with the piecewise smooth signals and several works have used it to exploit the smoothness of the depth map [6]–[10].

Data driven solutions are an alternative approach to address SR tasks and include dictionary learning and deep learning methods. Dictionary learning methods rely on sparsity assumptions and try to learn the correlation between the LR and the HR space from a set of training image pairs [4], [11]. On the other hand, deep learning (DL) methods rely on large amounts of data to find a direct mapping from the LR observations to the HR ground truth. Pioneer DL designs for general image SR include SRCNN [12], which demonstrated the potential of CNN data-driven approaches, ESRGAN [13], which relied on generative adversarial neural networks, RESNET [14] with its variants (RESNET50, RESNET101 etc), which introduced residual learning, and U-net [15], which showed that skip connections are useful for recovering fine details. State-of-the-art DL models for depth map SR include a multi-scale fusion model proposed in [16] where the multi-scale guided features are obtained by a VGG-like neural network, a multi-modal attention-based fusion model [17], a high-frequency guidance network that employs the octave convolution [18], and the model presented in [19] which uses a guidance image only at the training stage.

Despite their impressive results, learning-based methods require large computational resources and datasets, which can limit their applicability in real-world scenarios. Another significant drawback is their lack of interpretability and explainability, i.e., we do not know what the DL model has learned. Model-based deep learning designs are an alternative approach that tries to bridge the gap between deep learning and model-based solutions [10], [20], [21].

In this paper, we follow a model-based approach for the solution of single depth map SR which also leverages the representation power of deep neural networks. Specifically, we consider a graphical representation of depth maps and address the depth map SR as an inverse problem. In order to incorporate prior knowledge, promote smoothness and preserve the structural integrity of the depth maps, we propose a regularization of the problem at a feature level. We extract latent features from LR depth maps using a pretrained neural network model, and apply graph regularization by computing a graph Laplacian using the extracted features. The estimated SR depth maps are obtained as a solution to the graph-regularized problem using the Alternating Direction Method of Multipliers (ADMM) algorithm. Although the proposed approach benefits from deep learning representation models, it does not use any kind of training. Experimental results demonstrate the superior performance of the proposed approach against single-modal (non-guided) and guided SR methods.

The remainder of the paper is organized as follows: Section II reviews the related work. Section III presents the mathematical formulation and the details of the proposed algorithm. Section IV provides the experimental setup, the datasets, the experiments and the results of the algorithm. Finally, Section V concludes the paper.

II. Related work

In graph-based representations, we assume that the signal is represented in the form of a weighted undirected graph $\mathcal{G}$ and similarities in the signal are expressed by the edges $\mathcal{E}$ and the respective weights encoded in the adjacency matrix W. Graph-based representations consist a powerful tool in computer vision as they capture complex image structures and relationships between pixels [9], [22]. In image processing, typically, the nodes of the graph correspond to image pixels and the graph Laplacian is used to express similarity constraints between the pixels. Spectral graph theory has been leveraged to design effective regularization strategies that enhance feature preservation and structural coherence in reconstructed images. Graph-based regularization has been incorporated both in model-based approaches [22] and deep learning designs [9], [10].

Concerning depth map reconstruction, a graph Laplacian model exploiting prior information about the depth image and the corresponding color image was formulated in [6]. Instead of constructing the graph with the pixels, the authors of [7] propose to construct the graph with a group of similar patches. In [8] the edge weight distribution of an area with sharp edges is considered to be a bimodal distribution. A reweighted graph Laplacian regularizer is proposed to preserve sharp edges and promote the bimodal distribution of edge weights. In [10] the graph Laplacian is learned from the data and the regularized problem is solved by a DL design in an end-to-end manner.

Building upon these foundations, we consider a graph-based representation of the depth maps and compute the graph Laplacian matrix using latent features. Following an approach similar to [10], we obtain the latent representations from a deep neural network model. Unlike [10], where the latent features and the inverse mapping are learned from the data in an end-to-end learning framework, we use a pretrained model to extract the features and estimate the HR depth map by employing the ADMM algorithm to solve the corresponding graph-regularized optimization problem. We compare our results with state-of-the-art ADMM-based methods performing single depth map SR with other types of regularization [23], [24]. We also compare with state-of-the art guided depth map SR methods that use a neural network as a graph regularizer [10]. Experiments show that our method yields the best results in terms of root mean squared error (RMSE) and visual quality.

III. Depth map Super-Resolution using the ADMM algorithm and graph-based regularization

A. Depth map super-resolution and graph regularization

Similar to image SR, depth map SR is an inverse imaging problem [5] and can be formulated as follows:

$$\mathbf{y} = A\mathbf{x} + \epsilon, \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^{N_1 N_2}$ is the vectorized form of the observed $N_1 \times N_2$ depth map (LR), $\mathbf{x} \in \mathbb{R}^{S^2 N_1 N_2}$ is the unknown HR depth map, assuming upscaling by a factor S, $A \in \mathbb{R}^{N_1 N_2 \times S^2 N_1 N_2}$ is the degradation matrix, and ϵ is the additive noise. An optimization approach for the solution of (1) can be formulated as:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \, [g(\mathbf{x}) + h(\mathbf{x})], \tag{2}$$

where $g(\mathbf{x}) = \|\mathbf{y} - A\mathbf{x}\|_2^2$ is the fidelity term that enforces consistency with the observed data, and $h(\mathbf{x}) = \lambda R(\mathbf{x})$ is a regularization term, with $R(\mathbf{x})$ a regularizer weighted by $\lambda > 0$.

In order to promote smooth upscaling and also preserve the structural information, we consider a graphical representation of a depth map and propose a regularizer that encodes the spatial relationships between pixels by employing the graph Laplacian matrix. Assuming that each pixel in the depth map corresponds to a node in a graph and connections between neighboring pixels are represented by edges, the graph Laplacian matrix is given by

$$L = D - W, \tag{3}$$

where W is the adjacency matrix of the graph and D is the degree matrix. Therefore, we define a regularizer of the form

$$R(\mathbf{x}) = \mathbf{x}^T L \mathbf{x}. \tag{4}$$

Our approach considers the construction of a Laplacian matrix based on the similarity between pixels on a latent space. Specifically, we deploy a deep feature extractor (see Fig. 1) to obtain features for every pixel of the depth map and encode the similarity between features using a Gaussian kernel. For each

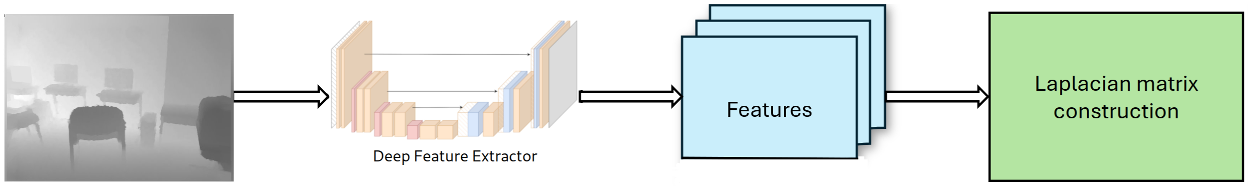

Fig. 1. Flowchart presenting the process of the Laplacian matrix construction. We pass the depth map into the deep feature extractor and construct the Laplacian matrix from the obtained features instead of traditionally using the depth map pixels.

pixel in the depth map, we consider its 4-connected neighbors, and the weights of the graph edges are determined as follows:

$$W_{ij} = \exp\left(-\frac{\|\mathbf{f_i} - \mathbf{f_j}\|_2^2}{\sigma^2}\right), \tag{5}$$

where $\mathbf{f_i}$ and $\mathbf{f_j}$ are the feature vectors with size equal to $classes$, and σ is a scaling parameter to control the sensitivity of the similarity to feature differences. For non-neighboring pixels we set $W_{ij} = 0$. As a deep feature extractor we employ a pretrained neural network that uses U-net [15] with ResNet50 encoder weights. We consider a bicubic interpolated depth map as input to the deep feature extractor, while the output features have dimensions of $classes \times SN_1 \times SN_2$, with $classes = 64$. After computing the adjacency matrix, the degree matrix D is calculated as:

$$D_{ii} = \sum_j W_{ij}. \tag{6}$$

Finally the Laplacian matrix is obtained from (3).

B. ADMM-based optimization

Employing ADMM for the solution of (2) results in Algorithm 1 [25]. We focus on Step 4 which involves the regularization term. The key challenge in this step is the

Algorithm 1 ADMM algorithm

1: **Input:** $\mathbf{u}^0 = 0$, $\mathbf{x}^0$, and $\rho > 0$
2: **for** $k = 1, 2, \ldots, t$ **do**
3: $\quad \mathbf{z}^k \leftarrow \text{prox}_{\rho g}(\mathbf{x}^{k-1} - \mathbf{u}^{k-1})$
4: $\quad \mathbf{x}^k \leftarrow \text{prox}_{\rho h}(\mathbf{z}^k + \mathbf{u}^{k-1})$
5: $\quad \mathbf{u}^k \leftarrow \mathbf{u}^{k-1} + (\mathbf{z}^k - x^k)$
6: **end for**
7: **Return:** $\mathbf{x}^t$

calculation of the proximal operator

$$\text{prox}_{\rho h}(\mathbf{v}) = \arg\min_{\mathbf{x}} \{\rho h(\mathbf{x}) + \frac{1}{2}\|\mathbf{x} - \mathbf{v}\|_2^2\}, \tag{7}$$

which is not straightforward and requires careful treatment due to the graph Laplacian L. We minimize (7) by setting the gradient to zero. Then, the calculation of the proximal operator induces to the solution of a quadratic minimization problem, which can be reformulated as a linear system:

$$(\rho I + 2\lambda L)\mathbf{x} = \rho \mathbf{v}, \tag{8}$$

where ρ is the penalty parameter for the ADMM algorithm and I is the identity matrix with shape $S^2N_1N_2 \times S^2N_1N_2$. This is a linear system of equations that can be efficiently solved using modern numerical methods, such as conjugate gradient descent. The matrix $(\rho I + 2\lambda L)$ is sparse, which makes these methods computationally feasible even for large-scale depth map SR tasks.

Our method integrates graph-based regularization directly into the ADMM framework and does not rely on neural network training techniques; therefore, it requires much fewer computations compared to learning based approaches and is significantly more time efficient. Moreover, we do not use complementary information from a guided color HR image; we only use a single depth map as input.

IV. Results and Evaluation

In this section, we evaluate the performance of the proposed framework for depth map SR that uses the Laplacian matrix as a regularizer and the ADMM algorithm for the solution of the corresponding optimization problem.

A. Experimental setup

We conduct experiments on two widely used depth map datasets, namely, DIML and NYUv2. DIML is a dataset which contains both indoor and outdoor real-world depth maps; we focus on the indoor depth maps. NYUv2 also contains indoor depth maps captured from RGB-D sensors.

For our experiments, we use depth map patches of size 256×256. We also rescale the input and output depth maps in the range $[0, 1]$. The pretrained model for extracting the features from the depth map input is a U-Net with encoder weights from ResNet50 and $classes = 64$. For $\times 4$ upsample, the parameters of the proposed ADMM method are set as follows: $\lambda = 1.61$, $\rho = 0.11$ and $\sigma = 2.71$. For the $\times 8$ upsample, we use $\lambda = 1.01$, $\rho = 0.11$ and $\sigma = 3.21$. These parameters were selected through tuning on a validation set. Specifically, we performed a grid search and evaluated performance using RMSE. The chosen values correspond to those that achieved the best performance on the validation set. We run the algorithm for a total of 15 iterations.

We use the Segmentation Models PyTorch library to obtain the features from U-Net, a library with pretrained neural networks for image segmentation based on PyTorch. For the code implementation, we used the Scientific Computational Imaging Code (SCICO) [27], a powerful open-source software framework designed for solving imaging and inverse problems using state-of-the-art computational algorithms. The hardware

TABLE I
COMPARISON OF METHODS ON NYUV2 AND DIML DATASETS IN TERMS OF RMSE

	Proposed	DnCNN-ADMM [26]	TV-ADMM [23]	BM3D-ADMM [24]	de Lutio et al. [10]
$\times 4$ NYUv2	**0.0154**	0.0188	0.0232	0.0273	0.0203
$\times 8$ NYUv2	**0.0265**	0.0338	0.0413	0.0666	0.0277
$\times 4$ DIML	0.0152	0.0161	0.0212	0.0242	**0.0127**
$\times 8$ DIML	0.0258	0.0303	0.0393	0.0617	**0.0187**

setup we used is cpu Intel i3 12100, 16 GB ram and MSI Geforce RTX 4060 graphics card.

B. Compared methods and results

To assess the effectiveness of our method, we compare it against state-of-the-art ADMM-based methods that use different regularizers. Specifically, we compare it with Plug and Play (PnP) methods like DnCNN-ADMM [26], which is a method integrating deep learning within ADMM, using the DnCNN denoiser for regularization. We also compare it with BM3D-ADMM [24], which leverages non-local self-similarities to remove noise and enhance textures, and TV-ADMM [23], which enforces smoothness in images by using total variation. We also compare against the guided depth map SR method presented in [10], which applies graph-based regularization using a deep learning framework. For visual comparison, we also include results from bicubic upsampling.

The metric we use to evaluate the performance of our method is the average root mean squared error (RMSE). The experiments involved 5030 samples from the DIML dataset and 600 samples from the NYUv2. Table I presents average results for $\times 4$ and $\times 8$ upsampling factors. As can be seen, the results indicate that our method outperforms all the single modal ADMM-based methods used as a baseline. On the NYUv2 dataset, it also outperforms the state-of-the-art DL method presented in [10] which employs a guidance HR color image. Since our method achieves better results than [10], it follows that it also outperforms all other methods used for comparison in [10], that is, DKN [28], FDKN [28], and FDSR [18]. In summary, on the NYUv2 dataset, our method achieves the lowest RMSE, outperforming all the other methods, while on the DIML dataset, our method performs slightly worse than [10] by a small margin in RMSE.

Note that our method does not include any kind of training, therefore, it has a lower overall computational cost. On the other hand, compared to the other ADMM variants our method has a slightly higher per-image processing time due to the computation of the Laplacian matrix.

While the numerical results provide an objective measure of performance, the visual quality is also a significant aspect for depth map SR. In Figure 2, we present the reconstructed depth maps for $\times 4$ and $\times 8$ upsampling factors and different methods. Specifically, we present (a) the HR color image, (b) the reference HR depth map patch, (c) the corresponding LR patch, and the reconstructed HR depth maps obtained with (d) bicubic interpolation, (e) BM3D-ADMM [24], (f) the method of de Lutio et al. [10], (g) DnCNN-ADMM [26], (h) TV-ADMM [23], and (i) our method.

As can be seen, our method demonstrates sharper edge preservation and better depth consistency in structured regions and object boundaries. We observe that the TV-ADMM method produces overly smoothed and blurry depth maps that lose lots of information of depth and also quality. BM3D and DnCNN produce better results compared to TV-ADMM but they also have the same problem, that is, the depth maps are smoother than expected and they do not have sufficient details. We also observe that BM3D tends to introduce residual noise in texture-heavy regions. The method of De Lutio et al. [10] which uses a guidance HR image outperforms the other baseline methods, likely due to the additional learned weights enhancing depth variations. However, our method reconstructs the image with finer details and fewer artifacts, and provides a good balance between sharpness and smoothness.

V. CONCLUSIONS

In this work, we addressed single depth map SR as an inverse problem and proposed a graph-based regularization that uses latent representations of the considered depth maps. Our method relies on a pretrained deep neural network model to estimate the regularizer and uses a mathematical formulation to solve the inverse problem by employing the ADMM algorithm. Therefore, our approach combines the advantages of both data-driven and model-based methods. Experimental results have shown that the proposed method outperforms other ADMM-based solutions as well as deep learning-based guided SR approaches.

REFERENCES

[1] Z. Song, J. Lu, Y. Yao, and J. Zhang, "Self-supervised depth completion from direct visual-lidar odometry in autonomous driving," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 8, pp. 11 654–11 665, 2021.

[2] J. Li, W. Gao, and Y. Wu, "High-quality 3D reconstruction with depth super-resolution and completion," *IEEE Access*, vol. 7, pp. 19 370–19 381, 2019.

[3] J. Xie, R. S. Feris, and M.-T. Sun, "Edge-guided single depth image super resolution," *IEEE Transactions on Image Processing*, vol. 25, no. 1, pp. 428–438, 2015.

[4] W. Xu, Q. Zhu, and N. Qi, "Depth map super-resolution via joint local gradient and nonlocal structural regularizations," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 32, no. 12, pp. 8297–8311, 2022.

[5] Z. Zhong, X. Liu, J. Jiang, D. Zhao, and X. Ji, "Guided depth map super-resolution: A survey," *ACM Computing Surveys*, vol. 55, no. 14s, pp. 1–36, 2023.

[6] Y. Zhang, Y. Feng, X. Liu, D. Zhai, X. Ji, H. Wang, and Q. Dai, "Color-guided depth image recovery with adaptive data fidelity and transferred graph laplacian regularization," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 30, no. 2, pp. 320–333, 2019.

[7] C. Yan, Z. Li, Y. Zhang, Y. Liu, X. Ji, and Y. Zhang, "Depth image denoising using nuclear norm and learning graph model," *ACM Transactions on Multimedia Computing, Communications, and Applications (TOMM)*, vol. 16, no. 4, pp. 1–17, 2020.

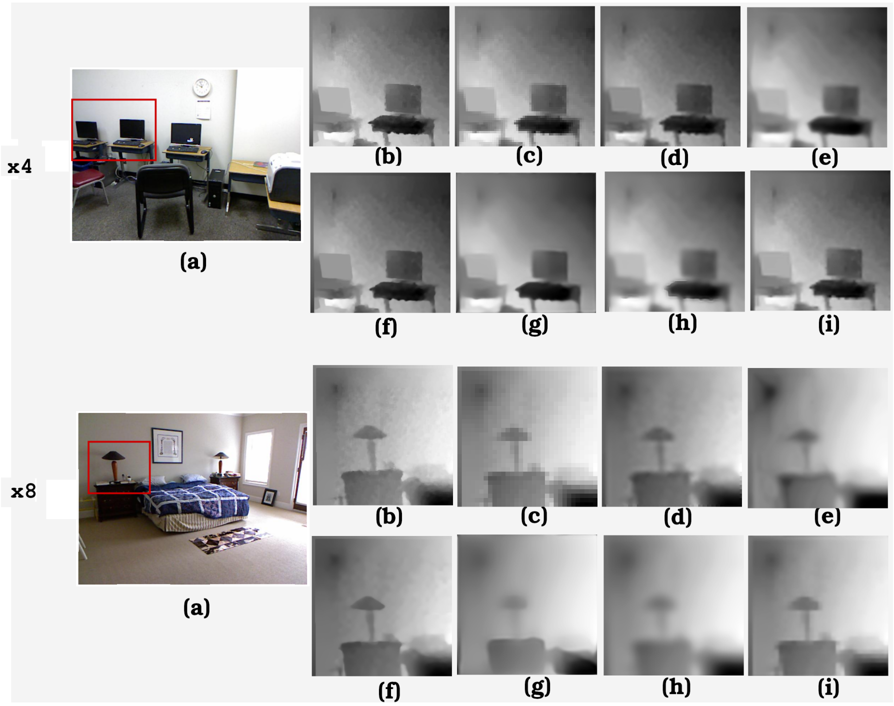

Fig. 2. Depth map SR results for upsampling factors $\times 4$ and $\times 8$. The reconstructed depth maps obtained with our method in (i) are compared with (d) bicubic interpolation, (e) BM3D-ADMM [24], (f) the method of de Lutio et al. [10], (g) DnCNN-ADMM [26] and (h) TV-ADMM [23]. We also present (a) the HR color image, (b) the ground truth and (c) the downsampled depth maps.

[8] J. Wang, L. Sun, R. Xiong, Y. Shi, Q. Zhu, and B. Yin, "Depth map super-resolution based on dual normal-depth regularization and graph laplacian prior," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 32, no. 6, pp. 3304–3318, 2021.

[9] B. Xu and H. Yin, "Graph convolutional networks in feature space for image deblurring and super-resolution," in *2021 International Joint Conf. on Neural Networks (IJCNN)*. IEEE, 2021, pp. 1–8.

[10] R. De Lutio, A. Becker, S. D'Aronco, S. Russo, J. D. Wegner, and K. Schindler, "Learning graph regularisation for guided super-resolution," in *Proc. IEEE/CVF Conf. on Comput. Vision and Pattern Recognit.*, 2022, pp. 1979–1988.

[11] J. Wang, W. Xu, J.-F. Cai, Q. Zhu, Y. Shi, and B. Yin, "Multi-direction dictionary learning based depth map super-resolution with autoregressive modeling," *IEEE Transactions on Multimedia*, vol. 22, no. 6, pp. 1470–1484, 2019.

[12] C. Dong, C. C. Loy, K. He, and X. Tang, "Learning a deep convolutional network for image super-resolution," in *Comput. Vision–ECCV 2014: 13th European Conf., Zurich, Switzerland, September 6-12, 2014, Proc., Part IV 13*. Springer, 2014, pp. 184–199.

[13] X. Wang, K. Yu, S. Wu, J. Gu, Y. Liu, C. Dong, Y. Qiao, and C. Change Loy, "Esrgan: Enhanced super-resolution generative adversarial networks," in *Proc. of the European Conf. on Computer Vision (ECCV) Workshops*, 2018, pp. 0–0.

[14] K. He, X. Zhang, S. Ren, and J. Sun, "Deep residual learning for image recognition," in *Proc. of the IEEE Conf. on Comput. Vision and Pattern Recognit.*, 2016, pp. 770–778.

[15] O. Ronneberger, P. Fischer, and T. Brox, "U-net: Convolutional networks for biomedical image segmentation," in *Med. Image Comput. and Comput.-Assisted Intervention–MICCAI 2015: 18th International Conf., Munich, Germany, October 5-9, 2015, Proceedings, part III 18*. Springer, 2015, pp. 234–241.

[16] T.-W. Hui, C. C. Loy, and X. Tang, "Depth map super-resolution by deep multi-scale guidance," in *Computer Vision–ECCV 2016: 14th European Conf., Amsterdam, The Netherlands, Oct. 11-14, 2016, Proc., Part III 14*. Springer, 2016, pp. 353–369.

[17] Z. Zhong, X. Liu, J. Jiang, D. Zhao, Z. Chen, and X. Ji, "High-resolution depth maps imaging via attention-based hierarchical multi-modal fusion," *IEEE Transactions on Image Processing*, vol. 31, pp. 648–663, 2021.

[18] L. He, H. Zhu, F. Li, H. Bai, R. Cong, C. Zhang, C. Lin, M. Liu, and Y. Zhao, "Towards fast and accurate real-world depth super-resolution: Benchmark dataset and baseline," in *Proc. IEEE/CVF Conf. on Comput. Vision and Pattern Recognit.*, 2021, pp. 9229–9238.

[19] B. Sun, X. Ye, B. Li, H. Li, Z. Wang, and R. Xu, "Learning scene structure guidance via cross-task knowledge transfer for single depth super-resolution," in *Proceedings of the ieee/cvf conference on computer vision and pattern recognition*, 2021, pp. 7792–7801.

[20] I. Marivani, E. Tsiligianni, B. Cornelis, and N. Deligiannis, "Multimodal deep unfolding for guided image super-resolution," *IEEE Transactions on Image Processing*, vol. 29, pp. 8443–8456, 2020.

[21] E. Tsiligianni, M. Zerva, I. Marivani, N. Deligiannis, and L. Kondi, "Interpretable deep learning for multimodal super-resolution of medical images," in *International Conf. on Med. Image Computing and Comput.-Assisted Intervention*. Springer, 2021, pp. 421–429.

[22] M. Rossi and P. Frossard, "Geometry-consistent light field super-resolution via graph-based regularization," *IEEE Trans. Image Process.*, vol. 27, no. 9, pp. 4207–4218, 2018.

[23] L. I. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Physica D*, vol. 60, no. 1-4, pp. 259–268, 1992.

[24] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image restoration by sparse 3D transform-domain collaborative filtering," in *Image Process.: Algorithms and Systems VI*, vol. 6812. SPIE, 2008, pp. 62–73.

[25] N. Parikh, S. Boyd *et al.*, "Proximal algorithms," *Found. Trends ® in Optimization*, vol. 1, no. 3, pp. 127–239, 2014.

[26] K. Zhang, W. Zuo, Y. Chen, D. Meng, and L. Zhang, "Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising," *IEEE Trans. Image Process.*, vol. 26, no. 7, pp. 3142–3155, 2017.

[27] T. Balke, F. Davis Rivera, C. Garcia-Cardona, S. Majee, M. T. McCann, L. Pfister, and B. E. Wohlberg, "Scientific computational imaging code (SCICO)," *Journal of Open Source Software*, vol. 7, no. LA-UR-22-28555, 2022.

[28] B. Kim, J. Ponce, and B. Ham, "Deformable kernel networks for joint image filtering," *Int. J. Comput. Vision*, vol. 129, no. 2, pp. 579–600, 2021.