Semi-Blind Channel Estimation and Near-Field Source Localization with Extremely Large Antenna Arrays

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***Abstract*—This paper proposes a low-complexity semi-blind solution for channel estimation when using Extremely Large-scale Antenna Arrays (ELAA), without prior knowledge of the considered channel model. More precisely, the new method combines pilots and a subspace approach for efficient and robust channel estimation, considering Intersymbol Interference (ISI). To reduce the costs, a Divide and Conquer (DAC) methodology is considered for a fast and parallelizable subspace estimation. For the positioning objective, physical localization parameters (i.e., angles and range) are accurately obtained using a simple least-squares fitting applied to the first channel tap (i.e. the shortest path), assuming a line of sight situation. A simulation-based study is provided to assess the potential of our semi-blind method.**

***Index Terms*—Near-field communications, semi-blind channel estimation, ELAA, mobile localization, Intersymbol Interference.**

I. INTRODUCTION

As a pivotal technology in 5G communications systems, large antenna arrays, known as massive MIMO, significantly boost transmission rates through efficient beamforming and precoding. To maximize the advantages of massive antennas, the latter are scaled to extremely large antenna arrays (ELAAs) for 6G communications, where the array aperture is substantially increased to enable ultra-high-speed communications. However, compared with massive MIMO, ELAA for 6G results in a fundamental change of the electromagnetic characteristics so that the radiation field can generally be divided into different regions (models) including far-field and near-field (NF) regions. Consequently, the channel estimation for ELAA highly depends on the adopted channel model (see [1]), making it vulnerable to scenario changes (e.g. near field, far field, hybrid). This dependency requires, in most cases, a prior estimation of the polar coordinates of the emitting source, leading to huge computational costs due to grid search and/or large matrix decompositions. Also, existing works on ELAA do not consider Intersymbol Interference (ISI) in their models for simplification [2–5], which would probably call for the use of Cyclic Prefix (CP) to try to address it at the expense of the useful throughput. However, with the targeted data rates in 6G (up to 1 Tbits/s), ISI could not be neglected. Moreover, only the predefined pilots are usually considered.

This paper first proposes an approach consisting of a channel estimation in a multi-path propagation scenario without specifying the channel's model and exploiting both the predefined pilots and the unknown data in a semi-blind scheme. Then, we propose a low-complexity algorithm for the estimation of the polar coordinates associated to the first (direct) path using a nonlinear least squares fitting approach with a linear prediction based technique for the initialization stage.

II. DATA MODEL AND PROBLEM FORMULATION

Let us consider a single antenna user located in the near-field region[1], transmitting T symbols to an antenna array with N elements.

Assume that the $N \times T$ baseband observation matrix, $\mathbf{Y}$, at the output of the antenna array, satisfies the following very common signal model:

$$\mathbf{Y} = \mathbf{HS} + \mathbf{W} \tag{1}$$

where $\mathbf{S}$ is an $L \times T$ Toeplitz matrix corresponding to the T transmitted symbols $\{s(t)\}_{0 \le t \le T-1}$ following L_0 different paths and a delay spread L, i.e.

$$\mathbf{S} = [\mathbf{s}(0), \cdots, \mathbf{s}(T-1)]$$

where $\mathbf{s}(t) = [s(t), s(t-1), \cdots, s(t-L+1)]^T$. $\mathbf{H} = [\mathbf{h}_0, ..., \mathbf{h}_{L-1}]$ is the $N \times L$ channel matrix with $\mathbf{h}_0$ representing the Line Of Sight (LOS) propagation channel. and $\mathbf{W} = [\mathbf{w}(0), \cdots, \mathbf{w}(T-1)]$ is the noise matrix whose columns are assumed random circular white Gaussian vectors with zero mean and covariance matrix $\sigma^2\mathbf{I}$. Let us assume that the first P transmitted symbols correspond to the predefined pilots, and the remaining symbols are the unknown data.

For the estimation of the localization parameters, we consider a scenario similar to the one adopted in [6] and [7], where a symmetric Uniform Linear Array (ULA) consisting of $N = 2M + 1$ elements with spacing d is used. But, instead of several sources transmitting each through an LOS path, we have one user transmitting through several paths[2], as depicted in Fig. 1. In this case, the channel between the

[1]Note that this information is not used for the channel estimation step but only for the mobile localization task.

[2]Here, for clarity of exposition, we consider the single-user case. Extension to a multi-user context offers additional insights that we will explore in subsequent work.

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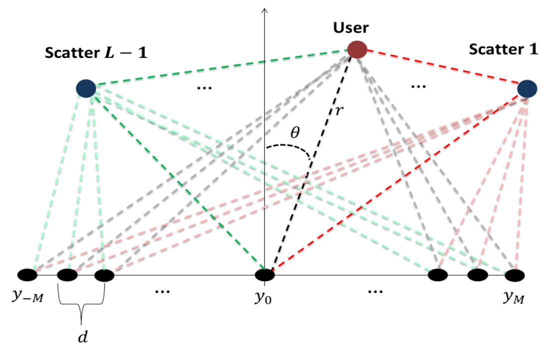


Fig. 1. Near-field multipath propagation.

m^{th} antenna, $m \in \{-M, \ldots, M\}$, and the l^{th} user/scatterer, $l \in \{0, \ldots, L_0 - 1\}$, located at $(r_l \cos\theta_l, r_l \sin\theta_l)$ is given by:

$$h_l(m) = \beta_l \exp(j\tau_{ml}) \tag{2}$$

where β_l denotes the channel complex gain[3] including the impact of the random reflection coefficient of the l^{th} scatterer as it was described in [11], and τ_{ml} is the phase delay of the received signal at the m^{th} antenna from the l^{th} user/scatterer with respect to the reference antenna ($m = 0$), given by [12]:

$$\tau_{ml} = \frac{2\pi}{\lambda}\left(\sqrt{r_l^2 + (md)^2 - 2mdr_l \sin\theta_l} - r_l\right) \tag{3}$$

where λ is the wavelength and (r_l, θ_l) are the polar coordinates of the l-th scatterer. By using a second-order Taylor expansion, τ_{ml} in equation (3) can be approximated as [6], [7]:

$$\tau_{ml} \approx m\omega_l + m^2\phi_l \tag{4}$$

where ω_l and ϕ_l are the electric angles defined by :

$$\omega_l \triangleq -\frac{2\pi d}{\lambda}\sin\theta_l, \qquad \phi_l \triangleq \frac{\pi d^2}{\lambda r_l}\cos^2\theta_l \tag{5}$$

In this context, mobile localization consists of estimating the coordinates (r_0, θ_0) of the first LOS path.

III. Proposed algorithm

A. First step: Channel estimation

The proposed algorithm aims to estimate, in a semi-blind scheme, the channel matrix for data detection without relying on a specific propagation model, to keep it robust to any scenario/model changes. In fact, the channel matrix $\mathbf{H}$ can be estimated as a specular one by estimating the locations of the scatterers and the fading parameters, but this is quite expensive and prone to modelization errors. It can also be estimated by the standard least-squares (LS) method using the pilots and the corresponding observation samples only, i.e. $\hat{\mathbf{H}}_P = \mathbf{Y}_P\mathbf{S}_P^{\#}$ (where $(.)^{\#}$ denotes the pseudo-inverse operator and index P refers to 'pilot'). This method requires many pilots, significantly reducing effective data throughput.

[3] In most papers in the literature, this gain is considered for simplicity as common to all antenna elements. [8] showed that it is quite true in the case of ELAA, where the range values are higher than $1.2D$, with D being the aperture of the array. However, in the case of a small number of elements, the gain should be considered as varying from antenna element to another due to the varying distance [9, 10].

Herein, we propose a semi-blind approach which would exploit both pilots and data for the channel estimation. The advantage, as will be illustrated later, is the reduction of the pilot size and improvement of the estimation accuracy. More precisely, we exploit the fact that, under the data model at hand, the subspace spanned by the channel matrix coincides with the one spanned by the L principal eigenvectors of the data covariance matrix [13] (or equivalently, the signal subspace spanned by the first L left singular vectors obtained from the singular value decomposition of $\mathbf{Y}$). This leads us to write the channel matrix as:

$$\hat{\mathbf{H}} = \mathbf{U}_s\hat{\mathbf{Q}} \tag{6}$$

where $\mathbf{U}_s$ is the $N \times L$ matrix whose columns span the signal subspace and $\hat{\mathbf{Q}}$ is an unknown $L \times L$ matrix that can be obtained from the pilots using the following LS criterion:

$$\hat{\mathbf{Q}} = \arg\min_{\mathbf{Q}} ||\mathbf{U}_s^H\mathbf{Y}_p - \mathbf{Q}\mathbf{S}_p||^2 = \mathbf{U}_s^H\mathbf{Y}_p\mathbf{S}_P^{\#} = \mathbf{U}_s^H\hat{\mathbf{H}}_P \tag{7}$$

As we can see, the pilots serve to estimate the $L \times L$ matrix $\mathbf{Q}$ instead of directly estimating the $N \times L$ matrix $\mathbf{H}$ which reduces significantly the number of parameters to be estimated, and consequently the number of needed pilots, since $L \ll N$. Note also that the estimated channel $\hat{\mathbf{H}}$ corresponds to the projection onto the signal subspace of $\hat{\mathbf{H}}_P$, i.e. $\hat{\mathbf{H}} = \mathbf{U}_s\mathbf{U}_s^H\hat{\mathbf{H}}_P$.

B. Divide and Conquer Strategy

In the ELAA case, the estimation of the signal subspace is prone to high computational complexity and even to bias due to the fact that the sample and antenna sizes might be of the same order, an adverse situation known in the literature as the high-dimension low-sample-size (HDLSS) context, e.g., [14]. To overcome this limitation, we propose to use a Divide and Conquer strategy, e.g. [15, 16], and adapt it in this specific situation to reduce the cost and get a parallelizable subspace estimation method. More precisely, let split the observation matrix in (1) into q submatrices as follows:

$$\mathbf{Y}_i = \mathbf{H}_i\mathbf{S} + \mathbf{W}_i, \qquad i = 1, \cdots, q \tag{8}$$

where, for simplicity, we assume $N = Kq$, K being an integer value satisfying $K = N/q > L$ and, using MATLAB notation, $\mathbf{Y}_i = \mathbf{Y}(i : q : \text{end}, :)$, $\mathbf{H}_i = \mathbf{H}(i : q : \text{end}, :)$.

Now, for every subsystem $1 \le i \le q$, one applies, in a parallel scheme if multiple computational units are available, our semi blind subspace estimation algorithm which leads to:

$$\begin{aligned} \hat{\mathbf{H}}_i &= \mathbf{U}_{s,i}\mathbf{U}_{s,i}^H\mathbf{Y}_{p,i}\mathbf{S}_p^{\#}, \quad i = 1, \cdots, q \\ &= \hat{\mathbf{H}}(i : q : \text{end}, :) \end{aligned} \tag{9}$$

where $\mathbf{U}_{s,i}$ is $K \times L$ matrix spanning the signal subspace of the i-th subsystem, and given by the first L left singular vectors of $\mathbf{Y}_i$. This DAC strategy allows us to reduce the computational cost by a factor q^2 and might improve the accuracy of the estimation in the HDLSS case.

An advantage of our channel estimation method, compared to a parametric one based on the specular channel model, is that we get the columns of $\hat{\mathbf{H}}$ sorted ascendantly with respect to the time delay of the path, so that the first column corresponds to the shortest path, considered as LOS, allowing simple angle and range estimation via the following fitting technique, applied to the first column of $\hat{\mathbf{H}}$.

C. Second step: Angle and range estimation

Here, we compute the correlation between the entries of the estimated channel vector $\hat{\mathbf{h}}_0$ such that we eliminate the quadratic terms in the phase, as is usually done in Linear Prediction (LP) based algorithms. Thus, from the channel model in (2) and the approximation in (4), one can express:

$$\begin{aligned}\mathbf{r}_1 &\triangleq [\hat{h}_0(-M)\hat{h}_0^*(M), \ldots, \hat{h}_0(-1)\hat{h}_0^*(1)]^{\mathrm{T}} \\ &\approx |\beta_0|^2[e^{j\frac{4\pi d}{\lambda}M\sin\theta_0}, \ldots, e^{j\frac{4\pi d}{\lambda}\sin\theta_0}]^{\mathrm{T}}\end{aligned} \quad (10)$$

from which, one can get an initial estimation of θ_0 as

$$\hat{\theta}_0 = \arcsin\left[\frac{\lambda}{4\pi d}\frac{1}{(M-1)}\sum_{k=1}^{M-1}\arg(r_1(k)r_1^*(k+1))\right] \quad (11)$$

where arg refers to the phase argument. It is worth noting that each term $\arg(r_1(k)r_1^*(k+1))$, for $k = 1, \ldots, M-1$, gives an estimate of the quantity given by $\frac{4\pi d}{\lambda}\sin\theta_0$, without ambiguity as long as $d \le \lambda/4$ and $\theta_0 \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. Here, the quarter-wavelength inter-element spacing would create a strong mutual coupling effect. However, its reduction, along with the reduction in cost and complexity through the use of sparse (coprime) array, will be presented in future work. Now, from (5) we get an initial estimation of the range r_0, using:

$$\hat{r}_0 = \frac{\pi d^2}{\lambda\hat{\phi}_0}\cos^2(\hat{\theta}_0) \quad (12)$$

where $\hat{\phi}_0$ is obtained by using vectors $\mathbf{r}_2$ and $\mathbf{r}_3$ given by

$$\begin{aligned}\mathbf{r}_2 &= [\hat{h}_0(-M+n)\hat{h}_0^*(M), \ldots, \hat{h}_0(-1)\hat{h}_0^*(n+1)]^{\mathrm{T}} \\ &\approx \alpha_2[e^{-j2M\gamma_1}, \ldots, e^{-j2(n+1)\gamma_1}]^{\mathrm{T}} \quad (13)\\ \mathbf{r}_3 &= [\hat{h}_0(-M)\hat{h}_0^*(M-n), \ldots, \hat{h}_0(-n-1)\hat{h}_0^*(1)]^{\mathrm{T}} \\ &\approx \alpha_3[e^{-j2M\gamma_2}, \ldots, e^{-j2(n+1)\gamma_2}]^{\mathrm{T}} \quad (14)\end{aligned}$$

with $\alpha_2 = |\beta_0|^2 e^{j(n\omega_0+n^2\phi_0)}$, $\alpha_3 = |\beta_0|^2 e^{-j(n\omega_0-n^2\phi_0)}$, $\gamma_1 = \omega_0 + n\phi_0$ and $\gamma_2 = \omega_0 - n\phi_0$. $\hat{\phi}_0$ is then computed as

$$\hat{\phi}_0 = \frac{1}{2n}(\hat{\gamma}_1 - \hat{\gamma}_2) \quad (15)$$

where $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are obtained from $\mathbf{r}_2$ and $\mathbf{r}_3$ as

$$\hat{\gamma}_1 = \frac{1}{2}\frac{1}{(M-n-1)}\sum_{k=1}^{M-n-1}\arg(r_2(k)r_2^*(k+1)) \quad (16)$$

$$\hat{\gamma}_2 = \frac{1}{2}\frac{1}{(M-n-1)}\sum_{k=1}^{M-n-1}\arg(r_3(k)r_3^*(k+1)) \quad (17)$$

ϕ_0 is usually too small to be directly estimated by other combinations between the entries of $\hat{\mathbf{h}}_0$. That is why (15) improves its estimation, thanks to the parameter n. An optimal choice of the latter is found experimentally in [7] as $n = \textit{round}\{0.6(M+1)\}$, where *round* denotes the integer round-off operation. Finally, we can refine the result with an iterative LS fitting method. For this, we define the objective function $J(r, \theta, \beta)$ and the associated optimization problem as:

$$(\hat{r}, \hat{\theta}, \hat{\beta}) = \arg\min_{r,\theta,\beta} J(r,\theta,\beta) = \arg\min_{r,\theta,\beta} ||\hat{\mathbf{h}}_0 - \beta\mathbf{b}(r,\theta)||^2$$

The gain β can be initialized as $\mathbf{b}(\hat{r}_0, \hat{\theta}_0)^{\#}\hat{\mathbf{h}}_0$, where $\mathbf{b}(r,\theta)$ is a steering vector constructed following the near-field model in (2). The parameters r and θ are then tuned by using few iterations of the well-known Levenberg-Marquardt's (LM) method, and jointly updated at iteration $i+1$ as:

$$\begin{pmatrix}\hat{r}_0^{(i+1)} \\ \hat{\theta}_0^{(i+1)}\end{pmatrix} = \begin{pmatrix}\hat{r}_0^{(i)} \\ \hat{\theta}_0^{(i)}\end{pmatrix} - \left(\gamma\mathbf{I}_2 + \mathbf{A}^{(i)}\right)^{-1}\begin{pmatrix}\bigtriangledown_r J^{(i)} \\ \bigtriangledown_\theta J^{(i)}\end{pmatrix} \quad (18)$$

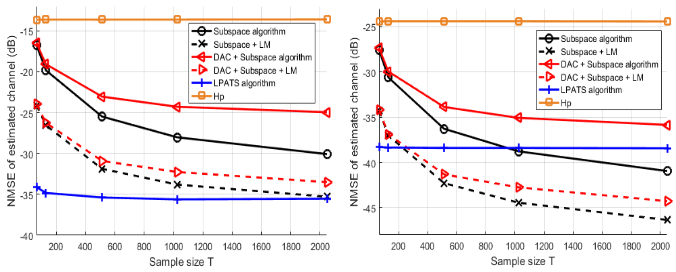
where γ is an arbitrary damping coefficient that helps to insure the non-singularity of the inverted matrix, $\mathbf{A}^{(i)}$ is the Hessian matrix of $J^{(i)}$, given by :

$$\mathbf{A}^{(i)} \triangleq \begin{pmatrix}\frac{\partial^2 J^{(i)}}{\partial r^2} & \frac{\partial^2 J^{(i)}}{\partial r\partial\theta} \\ \frac{\partial^2 J^{(i)}}{\partial\theta\partial r} & \frac{\partial^2 J^{(i)}}{\partial\theta^2}\end{pmatrix} \quad (19)$$

The gain β is updated as $\hat{\beta}^{(i+1)} = \mathbf{b}(\hat{r}_0^{(i+1)}, \hat{\theta}_0^{(i+1)})^{\#}\hat{\mathbf{h}}_0$.

IV. Simulation results

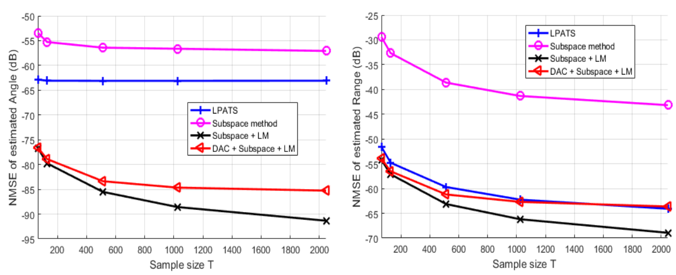
To assess the performance of our algorithm, we use for comparison the LP-based localization method called LPATS (LP Approach with Truncated SVD and oblique projection) [7], which shows better performance, at moderate to high SNRs, than other existing localization methods such as the weighted linear prediction method (WLPM) [6], the generalized ESPRIT and MUSIC based method (GEMM) [17] and the covariance approximation method for direction-finding (CAMDF) [18]. We omit here comparisons with pilot-based, polar-domain methods using grid search approach, e.g., [2], due to their high complexity and observed ill convergence when the initialization step is not accurate. Moreover, none of them consider ISI, for convenience. We performed simulations with fixed $P = 32$ pilot symbols, $L_0 = 3$ propagation paths with a delay spread $L = L_0$, $N = 357$ antenna elements, a carrier frequency $f_c = 50$ GHz, an inter-element spacing $d = \lambda/4$, and $q = 7$ submatrices for the DAC approach. The given results (statistics) are averaged over 500 Monte Carlo runs. The range values were chosen to be between Fresnel distance (FD) and Rayleigh distance (RD), i.e. $r_l \in [0.62\sqrt{D^3/\lambda}, 2D^2/\lambda]$, where $D = 2Md$ is the aperture of the array. In this work, we have set $(r,\theta) \in \{(5m, -10°), (40m, 0°), (70m, 20°)\}$ for the three considered paths, with $(\mathrm{FD} \approx 3.13m, \mathrm{RD} \approx 95.58m)$. Here, we evaluate the accuracy of the different methods using the normalized mean square error (NMSE). In addition, for the LP-based method, $\hat{\mathbf{H}}_P$ is used to retrieve the right order of the columns of $\hat{\mathbf{H}}$ and to estimate complex gains. In Fig. 2, the estimated channel NMSE is plotted versus the sample size T. As can be seen, at low SNR, subspace estimation is significantly improved with increasing T, but its accuracy remains lower than that of LPATS. However, by reconstructing the channel with the NF model as shown in III-C, when applied on each column of $\hat{\mathbf{H}}$ (referred to as Subspace + LM), we outperform LPATS as shown in Fig. 2 (b).



(a) SNR=0 dB (b) SNR=10 dB

Fig. 2. NMSE of the estimated channel versus sample size T

Fig. 3 shows that the subspace method outperforms the LPATS algorithm for localization purposes whatever T, mainly due to the Levenberg-Marquardt refinement algorithm. It is worth noting that the refinement step in the LPATS algorithm is based on oblique projection in the multipath case. When N is large, the LPATS algorithm shows a saturation effect, because of small bias due to Taylor's approximation in (4).



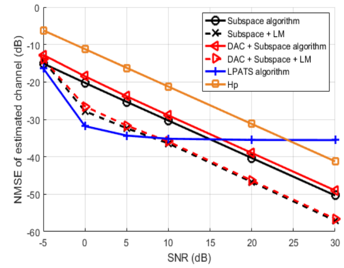
(a) estimated angle (b) estimated range

Fig. 3. NMSE versus sample size T for SNR = 0 dB

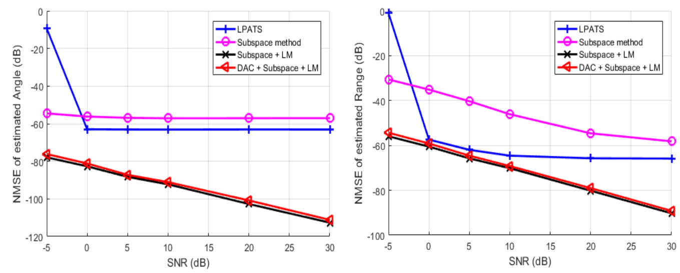
Fig. 4, corresponding to the NMSE of the estimated parameters versus the SNR, shows the improvement that we can get with the subspace method, even for low SNRs, despite its relative simplicity as compared to LPATS.

Finally, to test the robustness of the method, we set the scatterers in the same direction but at different locations $(\theta_1 = \theta_2 = 0°)$ and $(r_1, r_2) = (40, 70)m$. The corresponding results are shown in Fig. 5, where the subspace method still performs very well, whereas LPATS algorithm does not. This is due to the fact that our approach estimates first the channel independently of the positioning parameters, which are the estimated accurately using the LM technique. This is not the case of parametric methods in general, where any ambiguity in the location parameter estimation impacts the accuracy of the estimated channel.

Regarding computational costs, expressed in terms of the rough count of floating point operations (flops), the LPATS



(a) estimated channel



(b) estimated angle (c) estimated range

Fig. 4. NMSE versus SNR for $T = 256$ and $P = 32$

algorithm is made up of initialization and refinement steps, with about $\mathcal{O}[N^2T]$ and $\mathcal{O}[N^3N_iL]$ flops, respectively, for N_i (maximum) iterations in the second step. On the other hand, the truncated SVD used in the subspace method costs about $\mathcal{O}[NTL]$ flops in the worst case (without the reduction offered by the DAC approach). The LM algorithm, for its part, requires about $\mathcal{O}[NN_i]$ flops. Finally, we remind the reader that the estimation of the number of sources/paths is mainly based on matricial decomposition techniques [19], so our approach could exploit this decomposition whenever it is performed.

V. Conclusion

In this paper, we have proposed a new semi-blind approach for channel estimation and source localization in near-field communications with ELAA, considering ISI. When using a divide-and-conquer strategy, the proposed subspace approach is computationally competitive and relatively inexpensive compared to existing parametric methods. Moreover, our channel estimation method is 'robust' in the sense that it does not rely on a parametric propagation model in cross-field communications. After channel estimation, source localization is achieved with a simple LS fitting method.

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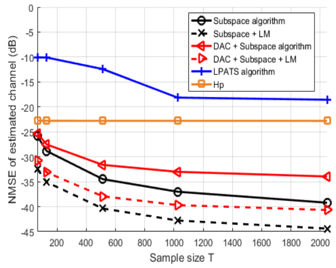
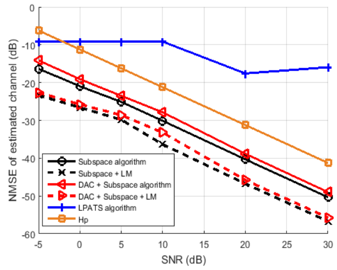
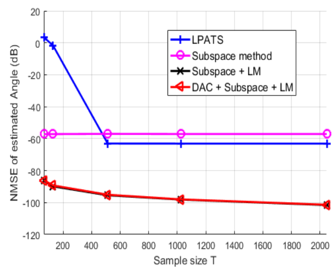


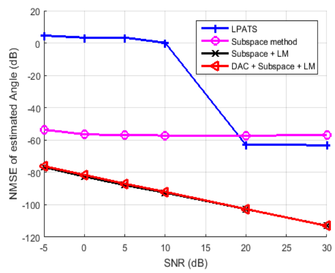
(a) Channel, SNR=10dB



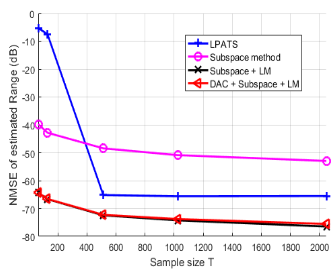
(b) Channel, $T = 256$



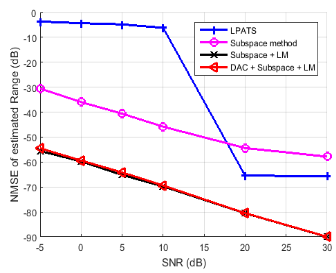
(c) Angle, SNR=10dB



(d) Angle, $T = 256$



(e) Range, SNR=10dB



(f) Range, $T = 256$

Fig. 5. Estimated parameters with $\theta_1 = \theta_2 = 0°$ and $P = 32$

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