Analysis of receiver implementation losses for GNSS signals

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***Abstract*—This paper presents an analysis of the implementation losses, L_{imp}, affecting a GNSS receiver. The implementation losses are the SNR loss at the correlator output due to payload and receiver imperfections with respect to an ideal case. L_{imp} is necessary to compute the link budget C/N_0, which must be compared to different C/N_0 thresholds to guarantee the correct processing of a GNSS signal. This paper provides a physical meaning as well as a mathematical definition of the 4 terms constituting the implementation losses; moreover, a computation scheme is also derived where the elements (signal noise, receiver) affecting each of the 4 terms are highlighted. Finally, this paper applies the theory to GPS L5, Galileo E5a and SBAS L5 signals for a civil aviation receiver; different signal definitions are considered (frequency bounding of the power spectrum density) and numerical values are calculated; 0.7dB can be expected for GPS and Galileo, and 1.27dB for SBAS, plus the accommodation of the miscellaneous loss term.**

Keywords—Implementation loss, C/N_0, filtering, correlation, quantization.

I. INTRODUCTION

The carrier-to-noise ratio, C/N_0, of a received Global Navigation Satellite System (GNSS) signal is a very important indicator since it is a sufficient metric to determine the GNSS receiver performance under nominal Additive White Gaussian Noise (AWGN) reception conditions and in the presence of interfering signals [1]. For example, the estimated C/N_0 is used to determine whether the tracking loops are locked by comparing them to theoretical C/N_0 thresholds or are used to weight the pseudorange measurements on the navigation filter [2]. The C/N_0 prediction from the link budget becomes thus very important to predict the performance of a GNSS receiver.

One key element of the C/N_0 prediction is the GNSS receiver implementation loss ratio or implementation losses. The receiver implementation losses are defined as the loss of signal-to-noise ratio, SNR, at the correlator output due to bandlimiting, quantization, sampling and other GNSS receiver imperfections as well as due to the transmitted signal (payload) imperfections [5][6][7]; remember that $C/N_0 \propto SNR$ [8]. The receiver implementation losses are modelled as a loss of the useful signal power, and are constituted of four terms, the correlation loss, the (receiver) quantization loss, the (receiver) bandlimiting and mismatch local replica loss and the miscellaneous loss [2][5]. In the literature, these terms are already defined and numerically derived although their interpretation and their relationship could be clarified. This clarification/interpretation is very important in civil aviation where the L5/E5a band C/N_0 link budget is very thin [2].

The objective of this article consists thus of presenting a comprehensive interpretation/tutorial of the implementation losses, of theoretically deriving them and of providing some numerical results for GPS L5, Galileo E5a and SBAS L5 signals. Other innovations are the mathematical definition of the correlation loss, the mismatch between the incoming signal and the local replica (especially for Galileo E5a) and the SARPs (Standards and Recommended Practices) definition of the signals' Power Spectrum Density (PSD).

This article is structured as follows. First, the general models of the received signal, the GNSS receiver and the SNR derivation are given. Second, the implementation loss term is defined and mathematically derived. Third, numerical results for the previously mentioned signals are calculated for a civil aviation receiver. Finally, this work is concluded.

II. GNSS SIGNAL, GNSS RECEIVER AND SNR MODELS

A. GNSS signal mathematical model

The received signal at the GNSS receiver antenna port, $r(t)$, is modelled as the addition of GNSS signals from different satellites, $s_m(t), l = 1 \dots M$, and the AWG noise, $n(t)$; M is the number of satellites:

$$r(t) = \sum_{l=1}^{M} s_l(t) + n(t) \quad (1)$$

The received GNSS signal from satellite l inside interval $[kT_I, (k+1)T_I)$ is modelled below; subindex l is removed from conciseness purposes as well as the modulated data.

$$s(t) = \sqrt{2P} c_{in}(t-\tau) \cos(2\pi(f_0 + f_d)t + \phi_0) \quad (2)$$

P is the received signal power, c_{in} is the Pseudorandom Noise (PRN) code signal, f_0 is the carrier frequency, τ is the propagation group delay, f_D is the Doppler frequency, ϕ_0 is the initial carrier phase and T_I is the coherent integration time.

One key point of (2) is the mathematical model of c_{in}. Although c_{in} is ideally defined in the SARPs [3] with the signal minimum power, P_{min}, the bandwidth containing this minimum power, B_{TX}, as well as the ideal chip modulation of the signal, the true transmitted signal can differ from the ideal signal description due to satellite l payload imperfections (modulation and filtering). Therefore, two additional signals can be defined, $c_{in}^{id}(t)$ which is the transmitted signal assuming an ideal chip modulation, and $c_{in,f}^{id}(t)$ which is the transmitted signal assuming an ideal chip modulation and assuming all the power is contained inside B_{TX}. The definition of these signals does not depend on the transmitting satellite.

$$c_{in,f}^{id}(t) = c_{in}^{id}(t) * h_{TX}^{id}(t)/\beta \quad (3)$$

$$H_{TX}^{id}(f) = \mathcal{F}\{h_{TX}^{id}(t)\} = \Pi(f/B_{Tx}) \quad (4)$$

$H_{TX}^{id}(f)$ is the ideal transfer function of the satellite payload filter, $\mathcal{F}$ is the Fourier Transform operation, $\Pi(x/Y)$

is the centered square function of variable x with duration Y, and β is a factor allowing to keep the same power in $c_{in,f}^{id}(t)$ as in $c_{in}^{id}(t)$ despite the filtering.

AWG noise, $n(t)$, and its PSD, $S_n(f)$, are modelled as shown in (5). $N(x, \sigma^2)$ represents a Gaussian distribution with mean x and variance σ^2.

$$n(t) \sim N(0, N_0/2) \quad S_n(f) = N_0/2 \tag{5}$$

B. GNSS receiver model and signal processing

A simplified model of the GNSS receiver containing only the essential blocks for calculating the implementation losses in section III is provided in Fig. 1. This figure includes the Radio-Frequency Front-End (RFFE) block as well as the correlator since the SNR is calculated at the correlator output.

The mathematical model of the received signal at the mixer output (not included in Fig. 1) is the same as the one given in expression (1), where the only difference is the carrier frequency, which is now equal to the intermediate carrier frequency, f_{IF}. Modelling the bandpass equivalent RFFE filter with bandwidth B with its impulse response, $h_{RF}^{B}(t)$, the GNSS signal, noise and their respective PSDs at the equivalent RFFE filter output are equal to:

$$r_{RF}(t) = r(t) * h_{RF}^{B}(t) \tag{6}$$

$$s_{RF}(t) = s(t) * h_{RF}^{B}(t) \quad S_s^{RF}(f) = S_s(f)|H_{RF}^{B}(f)|^2 \tag{7}$$

$$n_{RF}(t) = n(t) * h_{RF}^{B}(t) \quad S_n^{RF}(f) = S_n(f)|H_{RF}^{B}(f)|^2 \tag{8}$$

$$\sigma_{RF}^2 = B_{eq} N_0 \quad B_{eq} = 0.5 \cdot \int_{-\infty}^{+\infty} |H_{RF}^{B}(f)|^2 df \tag{9}$$

s_{RF} and S_s^{RF} are the GNSS signal and its PSD at the RFFE filter output, respectively, and n_{RF} and S_n^{RF} are the noise and its PSD at the RFFE filter output; $S_s(f)$ and $S_n(f)$ are the GNSS signal and noise PSD at the antenna port, and $H_{RF}^{B}(f)$ is the transfer function of the bandpass RFFE equivalent filter. σ_{RF}^2 is the n_{RF} power and B_{eq} can be defined as the one-sided equivalent RFFE noise bandwidth ($B_{eq} \geq B$).

The Automatic Gain Control (AGC) is assumed to multiply the signal by a gain, A_g, and the application of the ADC is assumed to sample the signal with sampling time, T_s, equal to inverse of the sampling frequency $F_s(= 1/T_s)$. At the Analog-to-Digital Converter (ADC) output, the quantized signal with b number of bits, $r_q[n]$, can be as expressed as [6].

$$r_q[n] = Q_b^{A_g}(r_{RF}[n]) \tag{10}$$

$$Q_b^{A_g}(x) = -(2^b - 1) + 2\sum_{i=-L}^{L} u(A_g x - i) \tag{11}$$

$Q_b^{A_g}$ is the b-bit quantizer operation with AGC gain equal to A_g, $u(t)$ is the Heaviside function and L is the maximum quantization level, $L = 2^{(b-1)} - 1$.

In [6], the quantizer impact on the GNSS signal and on the noise was modelled. The useful signal at the quantizer output, $s_q[n]$, is modelled by the multiplication of a constant factor to $s_{RF}[n]$; for the quantized noise, $n_q[n]$, the modeling of a nonlinear operation on a Gaussian noise is applied to $n_{RF}[n]$.

$$r_q[n] = s_q[n] + n_q[n] \quad s_q[n] = K_Q \cdot s_{RF}[n] \tag{12}$$

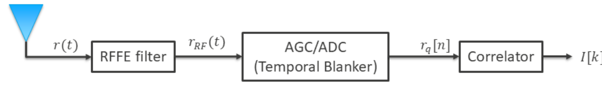

Fig. 1. Simplified model of a GNSS receiver

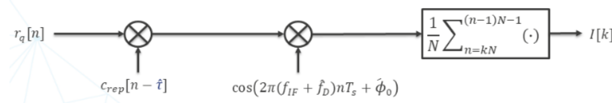

Fig. 2. In-phase correlator scheme

$$K_Q = \frac{2}{\sqrt{2\pi\sigma_{RF}^2}}\left(1 + 2\sum_{i=1}^{L} \exp\left(\frac{-i^2}{2A_g^2\sigma_{RF}^2}\right)\right) \tag{13}$$

$$S_n^Q(f) = \mathcal{F}\{R_n^Q[n]\} \tag{14}$$

$$R_n^Q[n] = sign(\rho_{RF}[n]) \frac{2}{\pi} \sum_{i=-L}^{L} \sum_{l=-L}^{L} I_{n,i,l} \tag{15}$$

$$I_{n,i,l} = \int_0^{|\rho_{RF}[n]|} \frac{\exp\left(-\frac{1}{2A_g^2\sigma_{RF}^2}\frac{i^2 + l^2 - 2ril}{1 - r^2}\right)}{\sqrt{1 - r^2}} dr \tag{16}$$

$$\rho_{RF}[n] = \frac{R_n^{RF}[n]}{R_n^{RF}[0]} \quad R_n^{RF}[n] = \frac{N_0}{2} \cdot \mathcal{F}^{-1}\{|H_{RF}(f)|^2\} \tag{17}$$

$R_n^Q[n]$ is the sampled autocorrelation function of $n_q[n]$, $R_{RF}[n]$ is the sampled autocorrelation function of $n_{RF}[n]$, and ρ_{RF} is its correlation coefficient. Since $S_n(f)$ is a flat constant value (irrespective of f), $R_{RF}[n]$ can directly be calculated from the inverse Fourier transform of $H_{RF}(f)$ square modulo.

C. SNR calculation at the correlator output.

The SNR at the correlator output is defined as the division of the useful signal power, P_s, by the power of the noise, P_n. To calculate its value, the code delay, Doppler frequency and initial carrier phase are assumed to be perfectly estimated, $\hat{\tau} = \tau$, $\hat{f}_D = f_D$ and $\hat{\phi}_0 = \phi_0$. Fig. 2 presents the in-phase correlator scheme.

$$SNR = P_s/P_n \tag{18}$$

Under these assumptions, P_s can be calculated from [7][8] where the dependence on the transmitted satellite l has been reintroduced:

$$P_s = P \cdot K_Q^2 \cdot K_f^2 \tag{19}$$

$$K_f = \int_{-Fs/2}^{Fs/2} H_{RF}(f)\bar{C}_{in}^{l}(f, f_{IF})\bar{C}_{rep}^{*}(f, f_{IF}) df \tag{20}$$

$$\bar{C}_x^y(f, f_{IF}) = \frac{1}{\sqrt{2}}\bar{C}_x^y(-f - f_{IF}) + \frac{1}{\sqrt{2}}\bar{C}_x^y(f - f_{IF}) \tag{21}$$

$\bar{C}_x^y(f, f_{IF})$ is the bandpass expression of $\bar{C}_x^y$, $\bar{C}_{in}^l(f)$ is the power-normalized baseband Fourier transform of the PRN code signal of satellite l, $c_{in}^l(t)$, $\bar{C}_{rep}(f)$ is the power-normalized baseband Fourier transform of the PRN code local replica signal, $c_{rep}(t)$, $*$ is the conjugated operator. F_s has been chosen to be large enough to avoid any significant impact of the sampling process on $\bar{C}_{in}^l(f)$ and on $\bar{C}_{rep}(f)$ calculations. To be completely accurate, $\bar{C}_{rep}(f)$ should be periodized with a period F_s as well as the multiplication result of $H_{RF}(f)\bar{C}_{in}^{l}(f)$ [7][8]; note that an antialiasing filter is usually implemented to remove this influence.

P_n can be calculated as in (22) ; N is the number of the correlator accumulated samples, $N \approx T_I/T_s$. $\bar{S}_{rep}(f)$ is the power-normalized PSD of the PRN code local replica signal.

$$P_n = \frac{1}{N}\int_{-Fs/2}^{Fs/2} S_n^q(f, f_{IF})\bar{S}_{rep}(f, f_{IF})df \qquad (22)$$

As well as before, to be completely accurate $\bar{S}_{rep}(f)$ should be made periodic. Note that $S_n^q(f)$ is already periodic by definition in (14).

III. Implementation Losses Definition

A. General Definition and model

The receiver implementation losses, L_{imp}, are defined as the loss of signal-to-noise ratio, SNR, at the correlator output due to the different imperfections of the GNSS receiver, imperfections of the transmitted signal (payload) as well as the mismatches between the ideal transmitted signal and the receiver local replica [2][5][7]. The implementation losses in the C/N_0 calculation are just modelled as a loss of power of the useful signal measured at the antenna port, P:

$$C = P \cdot L_{imp} \qquad (23)$$

The implementation losses can thus be calculated as shown in (24) where SNR_r is the SNR obtained when the previous defined imperfections and receiver configuration mismatches are considered, and SNR_{id} is the SNR obtained in ideal conditions and for an ideal receiver configuration. Upper index r and id also indicate *real* or *ideal* terms, and Δ_s and Δ_n indicate the useful power and noise variation between *real* and *ideal* situations.

$$L_{imp} = \frac{SNR_r}{SNR_{id}} = \frac{(P_s^r/P_s^{id})}{(P_n^r/P_n^{id})} = \frac{\Delta_s}{\Delta_n} \qquad (24)$$

As previously stated, despite its definition and thus the fact that the SNR degradation can occur due to a degradation of the useful signal power P_s and/or the noise power P_n, by convention, the implementation losses term is just modelled as an attenuation term impacting only P, the useful received signal power at the antenna port [5]. The mathematical model of the implementation losses, L_{imp}, is given below:

$$L_{imp} = L_T L_q L_m L_{misc} \qquad (25)$$

There are thus 4 terms modelling the implementation loss:

L_T is the correlation loss [2]: SNR degradation due to transmitted signal imperfections, limited to factors that cause correlation loss, in ratio, but not loss in signal power.

L_q is the quantization loss [2]: SNR degradation due to b-bits quantization and decimations.

L_m is the bandlimiting and local replica mismatch loss [2]: SNR degradation due to the effect of receiver band-limiting and the mismatch between the incoming and locally-generated modulations.

L_{misc} is the miscellaneous loss [2]: SNR degradation due to receiver filtering imperfections, in ratio, but not due to band-limiting or theoretical local replica mismatch.

Several important remarks must be made about (25). First, the names used to define the 4 different terms may differ from some names found in the literature, specially between L_T and L_m, where the name correlation loss sometimes was reserved for L_m [6]. In this work, we have used the last terminology used in civil aviation standardization groups [2] and also the term used in the GPS and Galileo signals Interface Control Documents (ICDs) [10][11]. Second, terms L_q and L_m are interdependent as shown in [5] and mathematically demonstrated in [6] and [7]. However, in this work, their impact is shown separately although the used mathematical models depend on each other. Third, in the literature, L_m term was only addressing the bandlimiting degradation; in this work, the additional degradation due to the voluntary generation of a local replica signal different from the ideal received signal is also addressed. Finally, each term of (25) can be mathematically derived by just calculating the SNR_r obtained with the imperfections or characteristics that are desired to model, and by dividing the result by SNR_{id}.

B. Ideal SNR determination

In this work, the determination of SNR_{id} is made by assuming that the maximum power a GNSS receiver will target to recover from the transmitted signal is the power defined in the SARPs [3] or system ICD [10][11]. This power is given in a predetermined bandwidth, B_{TX}. Therefore, for determining SNR_{id}, it is assumed first that the local replica is equal to the transmitted signal l and that the RFFE equivalent filter is a perfect square with bandwidth B_{TX}:

$$\bar{C}_{rep} = \bar{C}_{in}^{l} \qquad H_{RF}^{B}(f) = H_{TX}^{id}(f) \qquad (26)$$

Second, assuming an infinite-bit quantizer, its influence can be neglected on the received signal processing:

$$K_Q = 1 \qquad S_n^q(f) = S_n^{RF}(f) \qquad (27)$$

Third and last, the SNR_{id} is calculated by using (20) and (22) with the customizations given in (26) and (27).

$$SNR_{id} = \frac{P\int_{-B_{TX}/2}^{B_{TX}/2}\left|\bar{C}_{in}^{l}(f)\right|^2 df}{N_0\beta_0(B_{TX})/2N} \qquad (28)$$

$$\beta_0(B_{TX}) = \int_{-Fs/2}^{Fs/2}\left|H_{RF}^{B_{TX}}(f)\right|^2 \bar{S}_{rep}(f, f_{IF})df \qquad (29)$$

C. Correlation loss

To calculate the correlation loss, it is important to understand first the physical meaning of this term. In this work, this physical meaning is obtained from the correlation loss definition of GPS L5 ICD [10]: *"Correlation loss is defined as the difference between the SV* (Space Vehicle) *power received in the bandwidth defined in 3.3.1.1 (excluding signal combining loss) and the signal power recovered in an ideal correlation receiver of the same bandwidth using an exact replica of the waveform within an ideal sharp-cutoff filter bandwidth centered at L5, whose bandwidth corresponds to that specified in 3.3.1.1 and whose phase is linear over that bandwidth. The correlation loss apportionment due to SV modulation and filtering imperfections shall be 0.6 dB maximum."*

The interpretation made by the authors is that the correlation loss is caused by the satellite payload modulation imperfections as well as by the imperfections introduced by the payload filtering inside the bandwidth B_{TX}. The correlation term must thus consider the maximum SNR degradation due to the satellite payload imperfections of any satellite l, since each satellite transmits a different signal $c_{in}^{l}(t)$ (hardware imperfections differ among satellites). From this definition and interpretation, the correlation loss can be calculated by approximating K_f by two multiplying terms; K_f^l computes the useful power loss between the true transmitted signal from satellite l and the ideal transmitted signal, $c_{in,f}^{id}(t)$, and K_{ff} computes K_f (equation (*20*)) but assuming that the received signal is $c_{in,f}^{id}(t)$. Note that when $\bar{C}_{in}^{l}(f) = \bar{C}_{in,f}^{id,*}(f)$, equation (30) becomes equal to (20) since $K_f^l = 1$.

$$K_f \approx K_f^l \cdot K_{ff} \tag{30}$$

$$K_f^l = \int_{-Fs/2}^{+Fs/2} \bar{C}_{in}^{l}(f)\bar{C}_{in,f}^{id,*}(f)df \tag{31}$$

$$K_{ff} = \int_{-Fs/2}^{+Fs/2} H_{RF}^{B}(f)\bar{C}_{in,f}^{id}(f)\bar{C}_{rep}^{*}df \tag{32}$$

Finally, assuming a perfect local replica, $\bar{C}_{rep} = \bar{C}_{in,f}^{id}$, an ideal RFFE equivalent filter with $B = B_{TX}$ (see (26)) and an infinite-bit quantizer (see (27)), L_T becomes (33) when searching for the maximum degradation among all satellites:

$$L_T = \max_{l}\left\{\frac{\int_{-Fs/2}^{+Fs/2} \bar{C}_{in}^{l}(f)\bar{C}_{in,f}^{id,*}(f)df}{\int_{-B_{TX}/2}^{B_{TX}/2}\left|\bar{C}_{in}^{l}(f)\right|^2 df}\right\} \tag{33}$$

Note that the L_T allows to remove the satellite l influence from the implementation losses by considering the maximum satellite loss value. Moreover, from now on, the numerator term is removed from SNR_{id} (see (28)) since its influence has already been accounted for in (24). The new SNR'_{id} is thus:

$$SNR'_{id} = \frac{P}{N_0\beta_0(B_{TX})/2N} \tag{34}$$

In the literature, SNR'_{id} is calculated with $\beta_0(B_{TX} \to +\infty) = 1$ despite the mismatch with the previous L_T definition. To be consistent with the literature, the same assumption will be used in section IV.

D. Quantization loss and Band-limiting/local replica loss

As demonstrated in [6], the quantization loss, L_q, and the band-limiting and mismatch local replica loss, L_m, are interdependent since the power of the noise entering the quantizer, $n_{RF}(t)$, impacts the power of the quantized noise, $n_q(t)$, and the quantization process has an effect on the PSD of $n_q(t)$, used in (22), which is equivalent to a modification of the RFFE filter transfer function, $H_{RF}^{B}(f)$.

From [6] and using SNR'_{id} from (34) and (18) to (22), the joint impact of L_q and L_m, L_{qm}, is modelled assuming a RFFE filter equivalent bandwidth $\leq B_{TX}$. Moreover, using the individual expressions of L_q and L_m in [6], which are obtained for L_m when an infinite-bit quantizer is used and for L_q when a RFFE equivalent filter with an infinite bandwidth is used ($B \to +\infty$), L_{qm} can be expressed as:

$$L_{qm} = L_q^J \cdot L_m^J \quad L_q^J = \Delta_s^{qJ}/\Delta_n^{qJ} \quad L_m^J = \Delta_s^{mJ}/\Delta_n^{mJ} \tag{35}$$

$$\Delta_s^{qJ} = K_Q^2 \qquad \Delta_n^{qJ} = \frac{\int_{-Fs/2}^{Fs/2} S_n^{q}(f)df}{N_0/2} \tag{36}$$

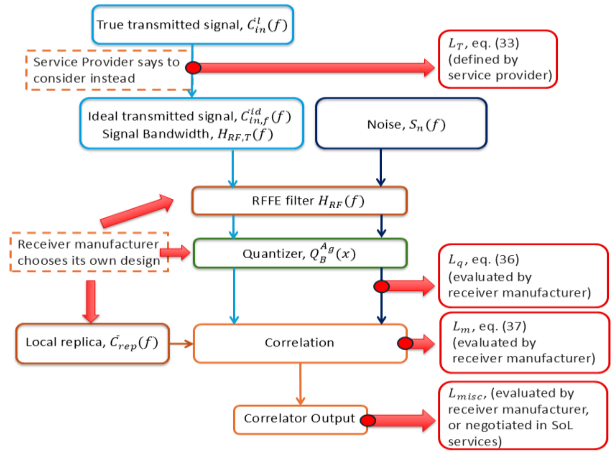

Fig. 3. Implementation loss scheme calculation

$$\Delta_s^{mJ} = K_f^2 \quad \Delta_n^{mJ} = \frac{1}{\beta_0(B_{TX})}\int_{-Fs/2}^{Fs/2} \bar{S}_n^{q}(f)\bar{S}_{rep}(f)df \tag{37}$$

Upper-index m or q specifies the type of loss, and upper-index J means joint calculation. The right term of (36) shows the influence of RFFE equivalent filter since $S_n^q(f)$ depends on $H_{RF}^{B}(f)$ as shown from (14) to (17). Moreover, the power of $n_q(t)$ is compared to the power obtained for the RFFE filter case with $B \to +\infty$, $B_{eq} = 1/2$. The right term of (37) shows the influence of the quantizer since $\bar{S}_n^q(f)$ depends on the modified $H_{RF}^{B}(f)$; if the quantizer was not implemented, ideally $\bar{S}_n^q(f) = |H_{RF}(f)|^2/B_{eq}$.

E. Miscellaneous loss

The computation of the miscellaneous loss term, L_{misc}, is not mathematically defined in the literature. This term refers to the SNR degradation due to the additional imperfections of the GNSS receiver. For the authors, these additional losses are due to the imperfect generation of the local replica, $c_{rep}(t)$, as well as the deviations of the designed $h_{RF}^{B}(t)$ ideal behavior with respect to the final implementation. Usually, this term is an accommodation provided to receiver manufacturers to account for the receiver's imperfections. Moreover, for Safety-of-Life (SoL) services, for example civil aviation, L_{misc} is negotiated in the standardization fora.

F. Summary

Fig. 3 presents a block scheme of the implementation loss calculation which summarizes the developments conducted during section III. In this figure, it can be observed at which step the loss term can be calculated; for example, although the SNR is always calculated at the correlator output, the correlation and quantization loss can be calculated without considering this operation. Moreover, the quantization loss only depends on the noise (no influence of the useful signal).

IV. Application to GNSS Civil Aviation GNSS receiver

In this section, the application of the implementation loss term calculation developed in previous section III for Dual-Frequency Multi-Constellation (DFMC) GNSS receivers for the L5/E5a band is conducted.

A. Received signal and GNSS receiver configuration

Three signals are inspected, GPS L5, Galileo E5a and SBAS L5. Each signal is defined assuming an infinite bandwidth ($B_{TX} = \infty$) and assuming the PSD definition given in the SARPS. TABLE I. presents the signals PSD SARPs bandwidth, B_{TX}, the percentage of power inside B_{TX} and the chip modulation PSD. Eq. (38) presents the Galileo E5a pilot power-normalized PSD derived from [9].

$$\bar{C}_{in}^{id}(f) = \sqrt{\frac{T_c}{1+1/\sqrt{2}} \frac{e^{-j\pi f T_c}}{2j\pi f} \cdot \frac{\cos(\pi f T_c)}{\cos(\pi f T_{sc}/2)}} \cdot \begin{bmatrix} \sqrt{2}\cos(\pi f T_{sc}/4) + (\sqrt{2}+2) - \cos(\pi f T_{sc}/2) \\ - \sin(\pi f T_{sc}/2) - \sqrt{2}\sin(\pi f T_{sc}/4) \end{bmatrix} \quad (38)$$

TABLE II. shows the DFMC GNSS receiver configuration. In this work, the selectivity requirement specified in [9] is used as the RFFE filter transfer function.

B. Numerical results

Fig. 4 presents the results for L_m and L_q when calculated jointly (J) and separated (S), for GPS/SBAS L5 (upper) and Galileo E5a (lower) with $B_{TX} \to \infty$. In Fig. 4 the theoretical results (solid lines) match the Monte Carlo simulation results (50s of simulation for a $T_I = 1ms$, shown as * in the figure) since they overlap. It can be observed that GPS and Galileo obtain the same results since term K_f is the same (and thus $L_m^S = -0.35dB$). K_f is different if a higher F_s is selected; for $F_s \to \infty$, then $L_m^S = -0.67dB$ for Galileo and $-0.41dB$ for GPS. Lowest $L_{imp}^J = -0.4dB$ is obtained for $A_g\sigma_{RF} = 1.6$.

TABLE III. presents L_T, L_q^J and L_m^J for GPS, Galileo and SBAS with $B_{TX} \to \infty$ and B_{TX} specified in TABLE I. with $A_g\sigma_{RF} = 1.6$. It can be seen that L_q^J improves the SNR (due to the increase of K_Q with respect to σ_Q^2), whereas L_f^J further decreases the SNR (due to $\bar{S}_n^q(f)$). SBAS L5 presents the worst case for two reasons. First, only 95% of its power is in B_{TX}, and thus $L_m^{j,SBAS}$ ($B_{TX} \neq \infty$) > $L_m^{j,GPS}$ ($B_{TX} \neq \infty$) . Second, L_T is much higher since its definition [3] does not bound B_{TX} and thus, in our opinion, the receiver bandlimiting effect is also counted in L_T. Finally, Fig. 5 presents $\bar{S}_n^q(f)$ for different values of $A_g\sigma_{RF}$. It can be observed that the quantizer indeed modifies the noise PSD originally shaped by $|H_{RF}^B(f)|^2$; the main consequence is found on the presence of a much higher noise floor (from $-146dB/Hz$ to $-94.5dB/Hz$ in the best case, 50dB increase) which increases Δ_n^{mJ} from the SSC term in (37). L_{imp} is calculated by adding columns 1, 3 and 4 from TABLE III. (without considering L_{misc}).

V. Conclusions

A complete analysis of the implementation loss term has been presented, providing a mathematical expression of the correlation losses, and detailing the interdependence between the quantization loss and the bandlimiting and mismatch local replica loss. Indeed, whereas the RFFE filter affects the noise power at the quantizer input, the quantizer changes the noise PSD which was originally shaped by the RFFE filter. It was seen that the main effect was to increase the noise floor by a minimum of $50dB$. Finally, GPS L5 and Galileo E5a have shown to have a L_{imp} equal to $0.7dB$ whereas SBAS is equal to $1.27dB$ plus an accommodation for the miscellaneous loss.

TABLE I. SARPs definition of GNSS signals PSD

GPS L5	Galileo E5a	SBAS L5
$\pm 12MHz$, 100%	$\pm 10.23MHz$, 100%	$\pm 12MHz$, 95%
BPSK(10)	Eq. (38)	BPSK(10)

TABLE II. DFMC GNSS Receiver Configuration

Quantizer	f_{IF}	RFFE filter		
		B	*Transition band*	*Floor*
3 bits	25MHz	20MHz	12.7MHz, 5.5dB/MHz	-70dB
Local Replica		BPSK(10)	F_s	100MHz

TABLE III. Implementation Losses results, $A_g\sigma_{RF} = 1.6$

	L_T	L_m^J ($B_{TX} = \infty$)	L_m^J ($B_{TX} \neq \infty$)	L_q^J
GPS L5	0.6	3.89	3.58	-3.48
Gal E5a	0.6	3.90	3.57	-3.48
SBAS L5	1	3.89	3.75	-3.48

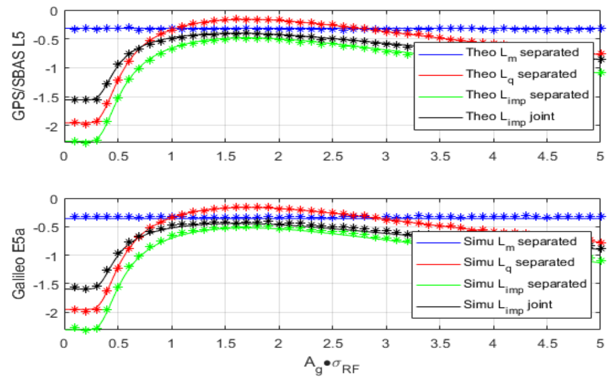

Fig. 4. L_m and L_q for GPS/SBAS L5 and Galileo E5a signals, $B_{TX} \to \infty$.

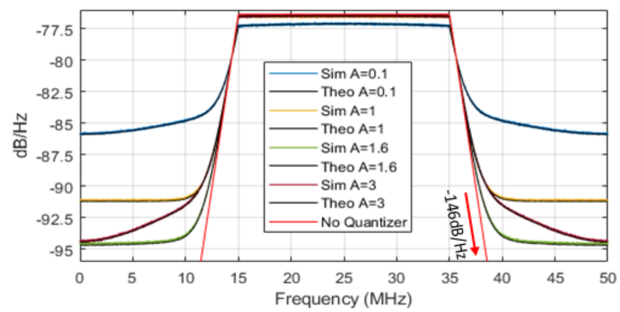

Fig. 5. Power-normalized noise PSD at the quantizer output

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