A radar-based positioning algorithm for the automotive scenario

1st Francesco Bandiera
Dipartimento di Ingegneria dell'Innovazione
Università del Salento
Lecce, Italy
francesco.bandiera@unisalento.it

2nd Giuseppe Ricci
Dipartimento di Ingegneria dell'Innovazione
Università del Salento
Lecce, Italy
giuseppe.ricci@unisalento.it

***Abstract*—Road safety applications for automotive scenarios rely on the ability to estimate the vehicles positions with high precision. Global navigation satellite systems (GNSS) and, in particular the global positioning system (GPS), are commonly used for self-localization, but especially in urban vehicular scenarios, due to obstructions, it may not provide the requirements of crucial position-based applications. In this paper, we investigate the potential of GPS-free positioning schemes and, in particular, we propose a localization algorithm in which each vehicle estimates its position exploiting range and azimuth radar measurements of an assigned set of landmarks with known position. At the analysis stage, we compare the performance to a Cramér-Rao lower bound (CRLB).**

***Index Terms*—Positioning, radar.**

I. Introduction

Road safety applications are emerging as an important feature of intelligent transportation systems (ITS). However, such applications pose numerous challenges for which ultimate solutions are still unavailable. One of the most important issues is how to guarantee accurate position information in the very diverse automotive scenarios [1], [2]. The global positioning system (GPS) is widely used for localization; however, as recent studies [3] show, the accuracy and availability of the GPS signal cannot always meet the requirements of crucial position-based applications. For instance, in dense urban environments, accuracy and availability of the GPS are limited by satellite visibility interruption, vehicle dynamics, and local errors (e.g., receiver noise and multipath) [4]. Preliminary research efforts, e.g., [5]–[7], have tackled this problem by focusing on standalone positioning systems that combine GPS data with additional measurements gathered from kinematic sensors available on board (Dead Reckoning, INS, etc.).

In recent years, vehicular ad-hoc networks (VANETs) [8] have been proposed by the automotive research community as a mean to realize a connected road environment where vehicles and infrastructure components can communicate to improve their location awareness [9]–[12]. In a VANET GPS-free positioning techniques can take advantage of the beacon packets transmitted from roadside units (RSUs), possibly in a cooperative fashion by jointly processing position-related data exchanged among a group of VANET nodes [13]. A receiver can exploit either range measurements based on received signal strength (RSS), time of arrival (TOA), and time difference of arrival (TDOA) [9], [14], [15] or angle of arrival (AOA) measurements [16]–[19] associated with signals transmitted from nearby anchors to determine its own position. Specifically, in [17] the authors investigated a vehicle-to-infrastructure (V2I) scenario where a vehicle equipped with an array of antennas is able to determine its position processing several AOA estimates collected by the vehicle along its trajectory; AOAs are obtained by the multiple signal classification (MUSIC) algorithm on the basis of packets broadcast by a road-side unit (RSU) in known position while the trajectory is reconstructed resorting to local INS measurements performed by the vehicle. This approach turns out to outperform GPS in urban environments. A cooperative tracking algorithm exploiting AOA measurements, obtained by processing beacon packets associated to vehicle-to-vehicle (V2V), in addition to V2I, communications, is investigated in [18]. In [19] localization based on several beacons is considered and an algorithm to estimate the AOAs in presence of mutual coupling and eventually the vehicle position is proposed; the limiting performance are assessed in terms of errors on the estimated AOAs.

Radar systems, a mature technology for remote sensing and surveillance, are widely used in a large variety of applications. In particular, radars have gained momentum for automotive applications, see [20] for an overview of state-of-the-art signal processing in automotive radars. Moreover, radar-based approaches to self-localization have been investigated for both indoor and outdoor scenarios [21]–[24]. [21] proposes a method that can be used in robots equipped with millimeter wave (mmWave) radars to estimate their position by taking advantage of the interference produced by other radars located in the same environment with well known position. The robot positions are computed using only the AOA of each radar interference. [22] provides an extensive performance analysis of an off-the-shelf mmWave radar sensor for people localization and tracking. In [24] the vehicle's position is inferred by association of landmark observations with map landmarks.

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Millimeter wave radars are a viable and low-cost solution, already available on vehicles, which have the potential to guarantee high accuracy localization at a low computational cost, also under adverse weather conditions. Finally, in [25] the limiting performance of a vehicle that estimates its position exploiting range and/or angle measurements of an assigned set of landmarks with known position is investigated.

In this paper, we focus on the derivation and performance assessment of a positioning algorithm based on measurements collected by radars mounted on a vehicle; the algorithm is capable of extracting from the set of detected targets the landmarks surrounding the vehicle and eventually determining the vehicle position. The achievable performance is compared with the Cramér-Rao lower bound (CRLB) calculated in [25]. The paper is organized as follows: next section describes the problem from a quantitative standpoint and introduces the proposed algorithm while Section III discusses performance of the algorithm. Conclusions are given in Section IV.

A. Notation

In the sequel, vectors are denoted by boldface letters, the acronym RV means random variable. Finally, we write $x \sim \mathcal{N}(m, \sigma^2)$ if x is a Gaussian RV with mean m and variance σ^2.

II. Problem Formulation and Algorithm Design

As previously mentioned, we are concerned with the problem of determining the position of a vehicle and we restrict our attention to localization in the planar case. We assume that the vehicle is equipped with a radar and that it is located at $P(x, y)$ within a given area somehow delimited by N landmarks located at $L_i(x_l(i), y_l(i)), h)$, $i = 1, \dots, N$, in a given Cartesian reference system. Of course, when the radar illuminates the environment, other unwanted phenomena emerge, such as unintended scatterers, noisy point clouds, multipath effects, range ambiguities, etc. Under the above assumptions, we suppose that the radar is able to measure the range and azimuth of the N landmarks of known positions and, in addition, of a certain number of unintended scatterers. We also assume that the heading of the vehicle is known and, for the sake of clarity, that it is aligned with the y-axis.

Assuming for the moment that the vehicle is able to select the landmarks (correct data association), we denote by $d(P, L_i)$ the distance of the radar to the ith landmark, i.e.,

$$d(P, L_i) = \sqrt{(x - x_l(i))^2 + (y - y_l(i))^2 + h^2} \qquad (1)$$

and by θ_i the angle formed by the projection of the line joining P and L_i onto the $x - y$ plane and oriented from P towards L_i, $d_{x-y}(P, L_i)$ say, and the y-axis, positive if measured counterclockwise,

$$\theta_i = \arcsin \frac{x - x_l(i)}{d_{x-y}(P, L_i)}, \qquad (2)$$

with

$$d_{x-y}(P, L_i) = \sqrt{(x - x_l(i))^2 + (y - y_l(i))^2}.$$

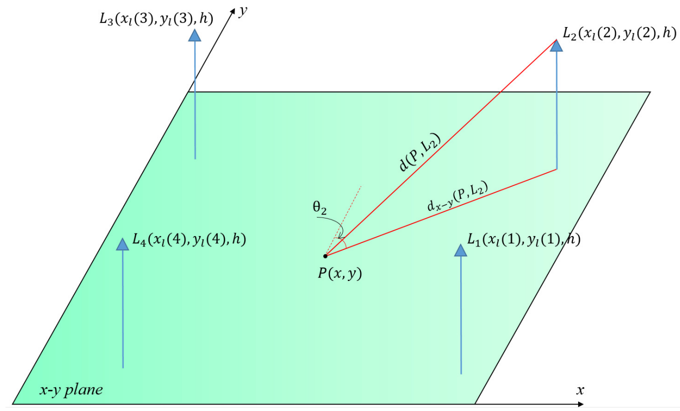

Fig. 1. An example of the positioning system with $N = 4$ landmarks. Vehicle is located at $P(x, y)$ and the angle θ_2 is measured on the $x - y$ plane.

A pictorial description of the system geometry is shown in Fig.1.

Moreover, we model the estimation errors as zero-mean Gaussian RVs; thus, the measurements of range and azimuth can be modeled as $R_i \sim \mathcal{N}(d(P, L_i), \sigma_r^2)$ and $\Theta_i \sim \mathcal{N}(\theta_i, \sigma_\theta^2)$ and, in addition, we suppose that the RVs $R_1, \Theta_1, \dots, R_N, \Theta_N$ are independent. It is important to stress that the Gaussian model for estimation errors of range and azimuth is widely adopted also due to its mathematical tractability [26]. However, other more accurate models could be considered; for instance, the von Mises distribution has been used to model AOA measurement errors in direction finding systems [27].

Recalling that we have assumed that the vehicle trajectory is parallel to the y-axis, it is possible to determine the vehicle position from the i-th north-east or north-west landmark using the formulas

$$\hat{x}_i = x_l(i) + R_i \sin \Theta_i,$$

$$\hat{y}_i = y_l(i) - R_i \cos \Theta_i$$

and, similarly, it is possible to compute the coordinates of the vehicle from the ith south-east or south-west landmark as

$$\hat{x}_i = x_l(i) + R_i \sin \Theta_i,$$

$$\hat{y}_i = y_l(i) + R_i \cos \Theta_i.$$

Strictly speaking, R_i is a measurement of $d(P, L_i)$, while correct estimation of the coordinates would require measurements (or estimates) of $d_{x-y}(P, L_i)$, i.e., the projected ranges. Since such estimates are not available or not immediately calculable, in this paper we use R_i as an approximation. Due to the fact that we are working with noisy measurements, observe that we now have N estimates of vehicle coordinates that are different from one landmark to another. In order to come up with the final, hopefully more accurate, estimate we propose to compute $\hat{x}$ and $\hat{y}$ as the sample mean of single estimates:

$$\begin{cases} \hat{x} = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_i, \\ \hat{y} = \frac{1}{N}\sum_{i=1}^{N}\hat{y}_i. \end{cases} \tag{3}$$

Alternatively, we can use the sample median of the $\hat{x}_i$s and $\hat{y}_i$s, $i = 1, \ldots, N$. To be more formal, let $\hat{x}_{(i)}$ and $\hat{y}_{(i)}$ the order statistics of $\hat{x}_i$ and $\hat{y}_i$, $i = 1, \ldots, N$. The estimator of the vehicle position can be written in this case as

$$\begin{cases} \hat{x} = \begin{cases} \hat{x}_{\left(\frac{N+1}{2}\right)}, & \text{if } N \text{ is odd} \\ \frac{1}{2}\left[\hat{x}_{\left(\frac{N}{2}\right)} + \hat{x}_{\left(\frac{N}{2}+1\right)}\right], & \text{if } N \text{ is even} \end{cases} \\ \hat{y} = \begin{cases} \hat{y}_{\left(\frac{N+1}{2}\right)}, & \text{if } N \text{ is odd} \\ \frac{1}{2}\left[\hat{y}_{\left(\frac{N}{2}\right)} + \hat{y}_{\left(\frac{N}{2}+1\right)}\right], & \text{if } N \text{ is even} \end{cases} \end{cases} \tag{4}$$

Up to this point, we have assumed that the true landmarks were correctly associated with the corresponding measurements. However, in practice, the output of a detector is a map of targets (each characterized by range, azimuth, and velocity) and the data association problem naturally arises due to the presence of other targets that could be mistaken for landmarks. To solve such problem we assume that the landmarks transmit their position and that the vehicle is also equipped with arrays of linear antennas (or more complicated configurations) to be used to estimate the AOAs of the signals transmitted by the landmarks. Such additional information can be exploited by the vehicle to identify the landmarks through comparison of their AOAs with azimuth measurements of the targets as viewed by the radars. The data association problem could also be solved by a joint processing of range and angle measurements. To this end, a coarse estimate of the vehicle position is necessary. Such estimate can be obtained from AOAs or, for vehicles not equipped with an array of antennas, from the GPS. The analysis conducted in the next section only assumes a selection of the landmarks based on comparison of angular measurements.

III. PERFORMANCE ANALYSIS

In this section we use Monte Carlo (MC) simulation to assess the performance of the proposed radar-based self-positioning algorithm based on range and azimuth estimates of the landmarks' positions. To this end, we assume $N = 4$ landmarks placed as follows: $L_1(-d, 0, h)$, $L_2(d, 0, h)$, $L_3(-d, -100, h)$, and $L_4(d, -100, h)$; the height with respect to the radar is $h = 2.5$ m. The vehicle moves over a uniformly sampled trajectory parallel to the y-axis, with sampling rate 1 m; more precisely, we consider the positions of a vehicle along a straight line of length 80 m, starting at $x = x_0$ m and $y = -90$ m and ending at $x = x_0$ m and $y = -10$ m. At each MC run we also add $N_F = 4$ "false landmarks" (one false landmark per true landmark). The positions of such false landmarks are randomly chosen with uniform distribution within squares of side $S = 10$ m centered on the positions of the true landmarks. The height of false landmarks is set to $h_F = 3.5$ m. Azimuth and range radar measurements from true and false targets are generated as independent and normally distributed RVs with mean equal to the corresponding true value and standard deviation $\sigma_r = 1$ m in range and $\sigma_\theta = 2°$ in azimuth; AOA measurements are also independent and normally distributed RVs with mean equal to the corresponding true value and standard deviation $\sigma_\theta = 2°$. The number of MC trials is set to 10^5.

Data association is handled in the following way. Let $\mathbf{\Phi} = [\phi_1 \cdots \phi_n]$ be the vector containing AOAs measured by the array mounted on the front of the vehicle[1] and let $\mathbf{\Gamma} = [\gamma_1 \cdots \gamma_{n+n_F}]$ the vector of the azimuthal values measured by the radar for the same angular region; for every i from 1 to n, we set $\Theta_i = \gamma_j$, with

$$j = \arg\min_i |\phi_i - \tilde{\mathbf{\Gamma}}|, \tag{5}$$

where $\tilde{\mathbf{\Gamma}}$ is the vector $\mathbf{\Gamma}$ purged from the values that have already been associated. An algorithmic representation is reported in the following table. Similar considerations apply to measurements collected by the array mounted on the rear of the vehicle and the "corresponding radar measurements."

Algorithm: Data Association procedure

Input : $\mathbf{\Phi} = [\phi_1 \cdots \phi_n]$, $\mathbf{\Gamma} = [\gamma_1 \cdots \gamma_{n+n_F}]$
Output: $\mathbf{\Theta} = [\Theta_1 \cdots \Theta_n]$
$\tilde{\mathbf{\Gamma}} = \mathbf{\Gamma}$;
for $i = 1 : n$
  $j = \arg\min |\mathbf{\Phi}(i) - \tilde{\mathbf{\Gamma}}|$;
  $\mathbf{\Theta}(i) = \mathbf{\Gamma}(j)$;
  $\tilde{\mathbf{\Gamma}}(j) = \text{NaN}$;
end

In a first set of examples we show the performance of the proposed positioning algorithm in terms of root mean square (RMS) estimation error for x and y in case of perfect data association, also in comparison to the CRLB for unbiased estimators derived in [25]. More precisely, in Figs. 2 and 3 we plot the RMS value for $d = 10$ m, with $x_0 = 0$ m and $x_0 = 8$ m, respectively. Inspection of the figures shows that the value of x_0 does not influence the performance for the considered system parameters. We also observe that estimation along y-axis is constantly under 1 meter and it is close to the CRLB, while the RMS error along the x-axis is greater, with the sample mean to be preferred in the center of the area and the sample median towards the borders.

In a second set of examples we show the performance in the case where the data association rule given by (5) is used. Figs. 4 and 5 report the corresponding RMS estimation errors, the remaining parameters are the same of Figs. 2 and 3, respectively. It is seen that data association produces a performance degradation of about 0.5 m on the y-axis (black

[1] We assume that there are not other systems transmitting on the same channel used by landmarks to communicate their position.

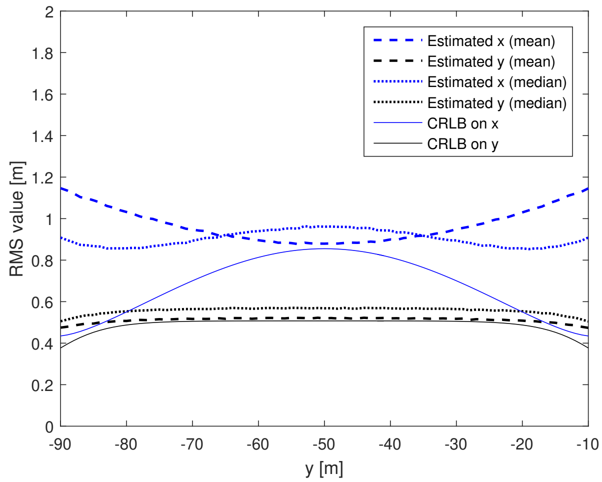

Fig. 2. RMS estimation error curves for $x_0 = 0$ m and perfect data association. Blue lines: error on the x-axis, black lines: error on the y-axis. Dashed lines: error for estimator given by (3), dotted lines: error for estimator given by (4).

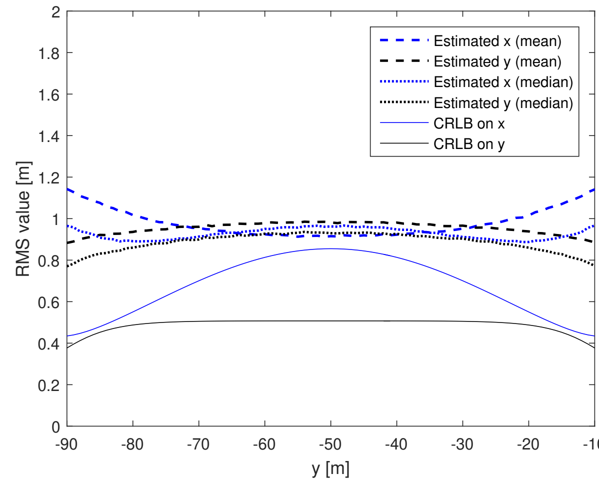

Fig. 4. RMS estimation error curves for $x_0 = 0$ and data association handled according to eq. (5). Blue lines: error on the x-axis, black lines: error on the y-axis. Dashed lines: error for estimator given by (3), dotted lines: error for estimator given by (4).

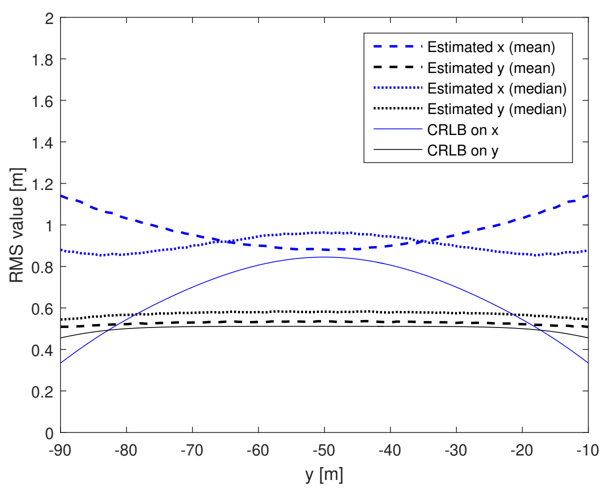

Fig. 3. RMS estimation error curves for $x_0 = 8$ m and perfect data association. Blue lines: error on the x-axis, black lines: error on the y-axis. Dashed lines: error for estimator given by (3), dotted lines: error for estimator given by (4).

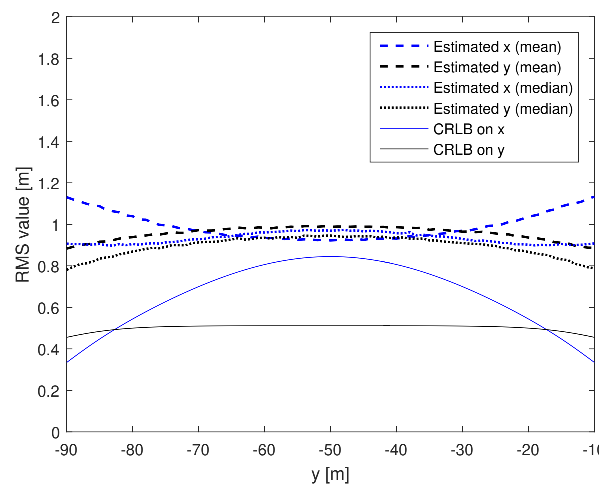

Fig. 5. RMS estimation error curves for $x_0 = 8$ and data association handled according to eq. (5). Blue lines: error on the x-axis, black lines: error on the y-axis. Dashed lines: error for estimator given by (3), dotted lines: error for estimator given by (4).

lines) while it does not affect too much the performance on the x-axis (blue lines). More generally, estimator based upon the sample median (4) provides a slightly better performance with respect to the estimator based upon the sample mean (3), this is likely due to the fact that the sample median is more robust versus isolated wrong associations (outliers).

Finally, we present some results about the estimated vehicle trajectory. In Fig. 6 we show an excerpt of the average of the estimated trajectories for estimators given by (3) and (4), together with the true trajectory. Inspection of the figure shows that both estimators could provide a biased estimate when the vehicle is far from the x center of the simulated area. Even tough it is a matter of a few centimeters, the problem of biasedness of the estimators is worth considering.

IV. Conclusions

In this paper, we have studied the potential of GPS-free positioning schemes for the automotive scenario and, in particular, we have proposed a localization scheme in which each vehicle estimates its position exploiting range and azimuth radar measurements of an assigned set of landmarks with known position. The analysis, conducted also in comparison to a Cramér-Rao lower bound, shows that the data association problem may represent a critical issue with a non-negligible impact on the achievable performance. For this reason, ongo-

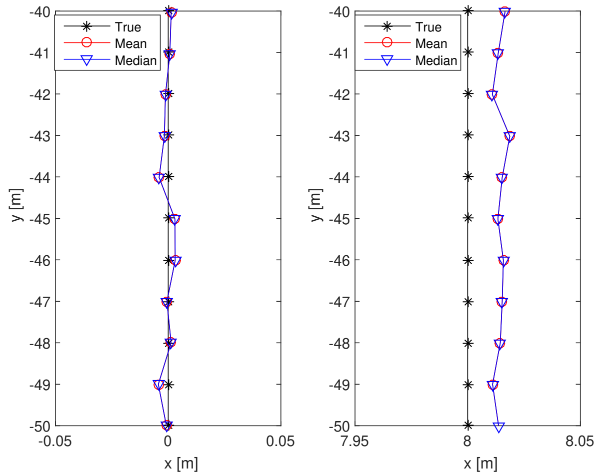

Fig. 6. Average estimated and true trajectories: $x_0 = 0$ m (left) and $x_0 = 8$ m (right). Data association handled according to (5).

ing research activity is oriented towards investigation of more sophisticated data association rules.

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