# An Unambiguous Hole-Filling method for Large-Baseline Distributed Nested Arrays Using Auxiliary Array

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***Abstract*—Distributed nested arrays with large baseline can significantly enhance array aperture with only a few elements. However, such spacing often results in increased sidelobe levels and angular ambiguities. To address these issues and achieve ambiguity-free DOA estimation, this paper proposes a subspace algorithm based on inserting contiguous auxiliary arrays. When the distributed nested array structure is set, we analyze the symmetric hole range, and derive a closed-form solution for both the required minimum number of auxiliary elements and the optimal position where to insert the first auxiliary element. This strategy ensures complete coverage of the hole region and maintains the continuity of the differential virtual array. Numerical simulations demonstrate that the proposed algorithm effectively removes angular ambiguities and outperforms the conventional dual-scale ESPRIT approach, achieving estimation accuracy comparable to that of a ULA with an equivalent aperture while using fewer elements.**

***Index Terms*—distributed nested arrays, large baseline distance, continuous auxiliary array, Spatial smoothing**

## I. Introduction

Uniform linear arrays (ULA) are widely used in direction-of-arrival (DOA) estimation due to their simplicity and unambiguous angular resolution [1]. However, their aperture is inherently limited by the number of physical sensors, restricting the achievable degrees of freedom (DOFs) and resolution [2], [3]. To address this, nested arrays were proposed, leveraging a two-level sparse structure to generate an extended difference coarray (DCA) and significantly improve virtual aperture [4]–[6]. While nested arrays outperform ULAs, their aperture expansion capability remains constrained by the sub-array configuration, limiting their applicability in large-scale sensing scenarios [7]–[9].

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Distributed arrays, composed of multiple spatially separated sub-arrays, offer a promising solution to further extend the physical aperture [10], [11]. By deploying sub-arrays with large baseline distances, distributed configurations achieve unprecedented spatial resolution and DOFs. Among these, distributed nested arrays combine the advantages of nested sub-arrays and distributed geometries, enabling both dense sampling within sub-arrays and ultra-wide aperture expansion across sub-arrays [12], [13]. Nevertheless, the large baseline spacing between sub-arrays introduces nonuniform holes in the DCA. These holes degrade DOA estimation performance because of the insurgence of angular ambiguity, so posing a critical bottleneck for practical deployments [14], [15].

This paper proposes a method to eliminate DCA holes in distributed nested arrays through the insertion of continuous auxiliary elements. By analyzing the hole distribution and symmetry properties, we derive closed-form expressions for the optimal positions and minimum number of auxiliary sensors required to completely fill the holes. The reconstructed virtual array forms a continuous uniform linear structure, enabling unambiguous high-resolution DOA estimation.

## II. Signal Model about Distributed Nested Arrays

In this paper, the distributed nested array $\mathbb{S}_{\mathrm{DisNA}}$ is composed of two nested arrays $\mathbb{S}_{\mathrm{NA}}$ with the same structure, where subarray 1 is defined as $\mathbb{S}_{\mathrm{NA1}}$ and subarray 2 is defined as $\mathbb{S}_{\mathrm{NA2}}$.

Since $\mathbb{S}_{\mathrm{NA1}}$ and $\mathbb{S}_{\mathrm{NA2}}$ share the same structure, we illustrate the concept using $\mathbb{S}_{\mathrm{NA1}}$ as an example. $\mathbb{S}_{\mathrm{NA1}}$ consists of two parts, denoted as $\mathbb{S}_{\mathrm{NA1}_{\mathrm{sub1}}}$ and $\mathbb{S}_{\mathrm{NA1}_{\mathrm{sub2}}}$. Specifically, $\mathbb{S}_{\mathrm{NA1}_{\mathrm{sub1}}}$ is a dense ULA composed of $M_1$ elements with an inter-element spacing of $d_1 = \lambda/2$, where $\lambda$ denotes the wavelength of the incident signal. In contrast, $\mathbb{S}_{\mathrm{NA1}_{\mathrm{sub2}}}$ is a sparse uniform linear array comprising $M_2$ elements, with an inter-element spacing of $d_2 = (M_1 + 1)\, d_1$. Overall, $\mathbb{S}_{\mathrm{NA1}}$ contains a total of $2M$ elements, satisfying the relation $2M = M_1 + M_2$. According to the array configuration, the positions of the elements can be expressed as $z_i d_1, i =$

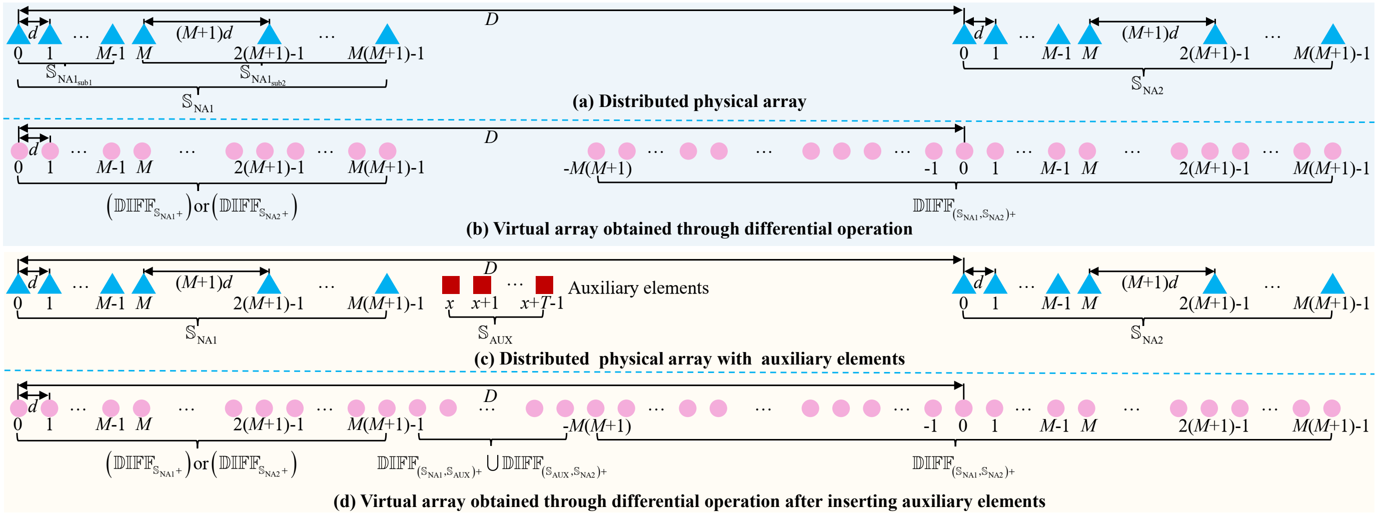


Fig. 1. Analysis of the array structure.

$1, 2, \cdots, 2M$, where $z_i \in \mathbb{S}_{\mathrm{NA1}}$, and $\mathbb{S}_{\mathrm{NA1}}$ denotes the set of positions in the nested array. The detailed structure is

$$\begin{aligned} &\mathbb{S}_{\mathrm{NA1}} = \mathbb{S}_{\mathrm{NA1}_{\mathrm{sub1}}} \cup \mathbb{S}_{\mathrm{NA1}_{\mathrm{sub2}}}, \\ &\mathbb{S}_{\mathrm{NA1}_{\mathrm{sub1}}} = \{0, 1, \cdots, M_1 - 1\}, \\ &\mathbb{S}_{\mathrm{NA1}_{\mathrm{sub2}}} = \{M_1, 2(M_1+1) - 1, \cdots, M_2(M_1+1) - 1\}. \end{aligned} \tag{1}$$

From Eq. (1), the array aperture of the nested array can be obtained as $\left(\bar{M} + 1\right) d_1$, where $\bar{M} = M_2(M_1+1) - 1$. According to Eq. (1), the positions of $\mathbb{S}_{\mathrm{DisNA}}$ is

$$\begin{aligned} &\mathbb{S}_{\mathrm{DisNA}} = \mathbb{S}_{\mathrm{NA1}} \cup \mathbb{S}_{\mathrm{NA2}}, \\ &\mathbb{S}_{\mathrm{NA1}} = \mathbb{S}_{\mathrm{NA}_{\mathrm{sub1}}} \cup \mathbb{S}_{\mathrm{NA}_{\mathrm{sub2}}}, \\ &\mathbb{S}_{\mathrm{NA2}} = D + \mathbb{S}_{\mathrm{NA1}}. \end{aligned} \tag{2}$$

where $D$ represents the baseline between the two sub-arrays. To better characterize the sub-array spacing, we use the array aperture of the nested sub-array as a reference length and define the baseline length between the distributed nested sub-arrays as $D = L \bullet \left(\bar{M} + 1\right)$, with $L$ denoting the ratio of the baseline length $D$ to the aperture of the nested subarrays. It is noteworthy that in this paper $M_1 = M_2 = M$. Then $D = L \bullet [M(M+1) - 1 + 1] = LM(M+1)$.

Consider $P$ narrow-band far-field sources incident on the distributed array. The received signal of the complete distributed nested array at time $t$ is

$$\mathbf{Y}(t) = \mathbf{AS}(t) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{A}$ is the $4M \times P$ dimensional array manifold matrix of the distributed nested array, $\mathbf{S}(t)$ is the $P \times 1$ dimensional source data, $\mathbf{n}(t)$ is the $4M \times 1$ dimensional noise vector at time $t$, and $\mathbf{Y}(t)$ is the $4M \times 1$ dimensional snapshot data vector. Then

$$E\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] = \sigma_n^2 \mathbf{I}, \tag{4}$$

where $\sigma_{\mathrm{n}}$ is the noise power.

When the first element from $\mathbb{S}_{\mathrm{NA1}}$ is used as the reference element, the manifold vector of the distributed nested array is represented as

$$\mathbf{a}(\theta_p) = \mathbf{a}_D(\theta_p) \otimes \mathbf{a}_1(\theta_p), \tag{5}$$

where

$$\begin{aligned} \mathbf{a}_1(\theta_p) = \big[&1, e^{jd\xi}, \cdots, e^{j(M-1)d\xi}, \\ &e^{jMd\xi}, e^{j[2(M+1)-1]d\xi} \cdots, e^{j[M(M+1)-1]d\xi}\big] \end{aligned} \tag{6}$$

is the manifold vector of reference subarray, $\xi = \frac{2\pi}{\lambda}\sin\theta_p$ and $\mathbf{a}_D = \left[\begin{array}{cc} 1 & e^{jD\xi} \end{array}\right]$. The covariance matrix is given by

$$\mathbf{R}_y = E\left[\mathbf{Y}\mathbf{Y}^H\right]. \tag{7}$$

## III. Problem Description

The data covariance matrix $\mathbf{R}_y$ is vectorized as

$$\mathbf{z} = \mathrm{vec}\left(\mathbf{R}_y\right) = \mathbf{BV} + \sigma^2\mathbf{e}. \tag{8}$$

where $\mathbf{B} = \left[\begin{array}{cccc} \mathbf{b}(\theta_1) & \mathbf{b}(\theta_2) & \cdots & \mathbf{b}(\theta_P) \end{array}\right]$ represents the manifold of the virtual array, and $\mathbf{V}$ is the vector of signal powers. In this formulation, $\mathbf{e} = \left[\mathbf{e}_1^T, \mathbf{e}_2^T, \cdots, e_{4M}^T\right]$, $\mathbf{e}_i \in \mathbb{R}^{4M\times 1}$ is the unit vector, $\mathbf{b}(\theta_p) = \mathbf{a}^*(\theta_p) \otimes \mathbf{a}(\theta_p)$ is the array manifold of the virtual array, whose elements correspond to the positions in the difference set of the sparse array elements—a structure known as the Difference coarray (DCA).

The manifold vector $\mathbf{z}$ can be expanded as

$$\mathbf{b}(\theta_p) = \left[\begin{array}{cccc} \zeta_1(\theta_p) & \zeta_2(\theta_p) & \cdots & \zeta_{4M}(\theta_p) \end{array}\right]^T, \tag{9}$$

with

$$\zeta_m(\theta_p) = \left[e^{(q_1 - q_m)\varphi_p}, e^{(q_2 - q_m)\varphi_p}, \cdots, e^{(q_{4M} - q_m)\varphi_p}\right]^T, \tag{10}$$

and $\varphi_p = -j2\pi d \sin\theta_p / \lambda$. The terms $(q_i - q_j)$ indicate the element positions in the DCA.

By removing redundancy in $\tilde{\mathbf{z}}$ and rearranging its elements in increasing order, one obtains $\tilde{\mathbf{z}}$ from a virtual array with an expanded coarray aperture, modeled as

$$\tilde{\mathbf{z}} = \tilde{\mathbf{B}}\tilde{\mathbf{V}} + \sigma^2\tilde{\mathbf{e}}. \tag{11}$$

where $\tilde{\mathbf{B}}$ is the manifold matrix for $\tilde{\mathbf{z}}$ and $\tilde{\mathbf{V}}$ represents the virtual source signal from a single snapshot. Select the continuous part of $\tilde{\mathbf{z}}$ and record it as $\bar{\mathbf{z}}$. The correlation matrix for the consecutive lags is then formed as

$$\mathbf{R}_{\bar{z}} = \bar{\mathbf{z}}\bar{\mathbf{z}}^H. \quad (12)$$

Due to the large baseline spacing in the sparse array, the obtained DCA still contains many holes (holes and nonconsecutive segments), which can hinder effective DOA estimation.

The differential virtual array shown in Fig.1 (b) can be obtained

$$\mathbb{DIFF}_{\mathbb{S}_{\text{DisNA}}+} = \mathbb{DIFF}_{\mathbb{S}_{\text{NA1}}+} \cup \mathbb{DIFF}_{\mathbb{S}_{\text{NA2}}+} \cup \mathbb{DIFF}_{(\mathbb{S}_{\text{NA1}},\mathbb{S}_{\text{NA2}})+} \quad (13)$$

where

$$\mathbb{DIFF}_{\mathbb{S}_{\text{NA1}}+} = \mathbb{DIFF}_{\mathbb{S}_{\text{NA2}}+} = \{0, 1, \cdots, M(M+1) - 1\} \quad (14)$$

$$\mathbb{DIFF}_{(\mathbb{S}_{\text{NA1}},\mathbb{S}_{\text{NA2}})+} = \{D + [-M(M+1) + 1, \cdots, -1, 0, 1, \cdots, M(M+1) - 2, M(M+1) - 1]\} \quad (15)$$

Hence,the hole range of the differential virtual array is

$$\mathbb{HOLE} = [M(M+1), D - M(M+1)] \quad (16)$$

If $\mathbb{DIFF}_{\mathbb{S}_{\text{DisNA}}+}$ is a continuous differential coarray with no holes, it must satisfy $M(M+1) - 1 \geq D - M(M+1) + 1$. Substituting $D = LM(M+1)$ into the inequality yields:

$$(L-2)M(M+1) \leq -2 \quad (17)$$

## IV. A hole-filling method based on the continuous auxiliary element

In this paper, we focus on designing distributed nested arrays with an inter-subarray spacing of $L \geq 5$ and do not extensively explore the case of $2 < L \leq 4$ for two main reasons:

① Design Complexity: For $L \leq 4$, reduced hole ranges and asymmetric DCA necessitate piecewise optimization of auxiliary element positions ($x$) and quantity ($T$). This case-specific approach lacks generalized closed-form solutions, complicating design scalability.

② Efficiency Tradeoff: Smaller baselines ($L \leq 4$) provide marginal virtual DOF gains at disproportionate ardware/computational costs. Larger baselines ($L \geq 5$) maximize aperture expansion, fully exploiting distributed nested array capabilities.

Thus, our work focuses on the generalized design for $L \geq 5$, to simplify the analysis and highlight the core innovations.

$\mathbb{S}_{\text{NA1}}$ and $\mathbb{S}_{\text{NA2}}$, are symmetrically distributed with a baseline distance $D$. Given that the hole range in the differential coarray has been derived in Eq. (16), the central positions of the holes are expressed as

$$C = \frac{\mathbb{HOLE}_{\text{start}} + \mathbb{HOLE}_{\text{end}}}{2} = \frac{D}{2} = \frac{LM(M+1)}{2} \quad (18)$$

where $\mathbb{HOLE}_{\text{start}} = M(M+1)$, $\mathbb{HOLE}_{\text{end}} = D - M(M+1)$.

As shown in Fig. 1 (c), assume that $T$ consecutive auxiliary elements are inserted at position $x$, an auxiliary array is

$$\mathbb{S}_{\text{Aux}} = \begin{bmatrix} x & x+1 & \cdots & x+T-1 \end{bmatrix} \quad (19)$$

To ensure complete coverage of $\mathbb{HOLE}$, the insertion $\mathbb{S}_{\text{Aux}}$ must make $\left\{\mathbb{DIFF}_{(\mathbb{S}_{\text{NA1}},\mathbb{S}_{\text{Aux}})+} \cup \mathbb{DIFF}_{(\mathbb{S}_{\text{Aux}},\mathbb{S}_{\text{NA2}})+}\right\} \supseteq \mathbb{HOLE}$ hold. Since $\mathbb{S}_{\text{Aux}}$ exhibits two distinct positional choices during the insertion process, the following sections will discuss each case separately.

### A. When $\mathbb{S}_{Aux}$ is on the left side, i.e., $x = x_1$

The difference cover between $\mathbb{S}_{\text{NA1}}$ and $\mathbb{S}_{\text{Aux}}$ spans the left half-section of $\mathbb{HOLE}$ the following holds

$$\begin{aligned} &\left[x_1 - (M(M+1) - 1), x_1 + T - 1\right] \\ &\supseteq \left[M(M+1) : C\right] \end{aligned} \quad (20)$$

The constraints are therefore given by

$$\begin{cases} x_1 - \big(M(M+1) - 1\big) \leq M(M+1), & (21a) \\ x_1 + T - 1 \geq C. & (21b) \end{cases}$$

The difference cover between $\mathbb{S}_{\text{NA2}}$ and $\mathbb{S}_{\text{Aux}}$ spans the right half-section of $\mathbb{HOLE}$ the following holds

$$\begin{aligned} &\left[D - (x_1 + T - 1),\ D + M(M+1) - 1 - x_1\right] \\ &\supseteq \left[C + 1 : D - M(M+1)\right] \end{aligned} \quad (22)$$

The constraints are therefore given by

$$\begin{cases} D - (x_1 + T - 1) \leq C + 1 & (23a) \\ D + M(M+1) - 1 - x_1 \geq D - M(M+1) & (23b) \end{cases}$$

Simplifying the right-endpoint Eq. (23b) yields

$$x_1 \leq 2M(M+1) - 1 \quad (24)$$

Regarding the left endpoint constraint Eq. (21b), which is $x_1 + T - 1 \geq C$, we substitute the expression for $C = \frac{LM(M+1)}{2}$ from Eq. (18). Rearranging this inequality to solve for $T$ yields:

$$\begin{aligned} x_1 + T - 1 &\geq \frac{LM(M+1)}{2} \\ \Rightarrow \quad T &\geq \frac{LM(M+1)}{2} - x_1 + 1. \end{aligned} \quad (25)$$

This equation (25) establishes a lower bound for $T$. The implication is that to find the minimum number of auxiliary elements, $T_{\text{min}}$, we must minimize the right-hand side of this inequality. Since $\frac{LM(M+1)}{2}$ and 1 are constants (for given $L$ and $M$), minimizing the right-hand side requires maximizing $x_1$.

When the equality holds, $T$ reaches to its minimum value, i.e., $T_{\min} = \frac{L}{2}M(M+1) - x_1 + 1$. When this occurs, $x_1$ should be the maximum value. Additionally, since $x_1 \leq 2M(M+1) - 1$ in Eq. (21a), it follows that

$$\begin{aligned} x_1 &= 2M\big(M+1\big) - 1, \\ T_{\min} &= \frac{L}{2}M\big(M+1\big) - \Big[2M\big(M+1\big) - 1\Big] + 1 \\ &= \frac{(L-4)M\big(M+1\big)}{2} + 2. \end{aligned} \quad (26)$$

Note that $T_{\min}$, representing the number of auxiliary elements, must be an integer. The derived expression for $T_{\min}$ in Eq. (26) naturally yields an integer. This is because $M(M+1)$ is the product of two consecutive integers, which is always an even integer. Consequently, $(L-4)M(M+1)/2$ is an integer (since $L-4$ is an integer).

### *B. When $\mathbb{S}_{Aux}$ is on the right side, i.e., $x = x_2$*

The difference cover between $\mathbb{S}_{\mathrm{NA1}}$ and $\mathbb{S}_{\mathrm{Aux}}$ spans the right half-section of $\mathbb{HOLE}$ the following holds

$$\begin{aligned}&\Big[x_2 - \Big(M(M+1)-1\Big),\ x_2+T-1\Big]\\&\supseteq [C+1 : D - M(M+1)]\,.\end{aligned} \tag{27}$$

The constraints are therefore given by

$$\begin{cases} x_2 - (M(M+1)-1) \le C+1, & (28a)\\ x_2 + T - 1 \ge D - M(M+1)\,. & (28b)\end{cases}$$

The difference cover between $\mathbb{S}_{\mathrm{NA2}}$ and $\mathbb{S}_{\mathrm{Aux}}$ spans the left half-section of $\mathbb{HOLE}$ the following holds

$$\begin{aligned}&\Big[D - \Big(x_2+T-1\Big),\ D + M\big(M+1\big) - 1 - x_2\Big]\\&\supseteq \big[\,M\big(M+1\big) : C\,\big]\,.\end{aligned} \tag{29}$$

The constraints are therefore given by

$$\begin{cases} D - (x_2+T-1) \le M(M+1)\,, & (30a)\\ D + M(M+1) - 1 - x_2 \ge C. & (30b)\end{cases}$$

Simplifying the left-endpoint Eq. (30a) yields

$$x_2 + T - 1 \ge (L-1)\,M\,(M+1) \tag{31}$$

Regarding the right endpoint constraint Eq. (28b)

$$\begin{aligned}&x_2+T-1 \ge (L-1)M(M+1)\\&\implies \quad T \ge (L-1)M(M+1) - x_2 + 1.\end{aligned} \tag{32}$$

When the equality holds, $T$ reaches to its minimum value, i.e., $T_{\min} = (L-1)M(M+1) - x_2 + 1$. When this occurs, $x_2$ should be the maximum value. Additionally, since $x_2 \le \left(\frac{L}{2}+1\right) M\,(M+1) - 1$ in Eq. (30b), it follows that

$$\begin{aligned}x_2 &= \Big(\frac{L}{2}+1\Big)M(M+1) - 1,\\ T_{\min} &= \frac{(L-4)M(M+1)}{2} + 2\end{aligned} \tag{33}$$

Similarly, the expression for $T_{\min}$ in Eq. (33) also naturally results in an integer value due to the aforementioned property that $M(M+1)$ is always even, ensuring the term $\frac{(L-4)M(M+1)}{2}$ is an integer.

In summary, given the values of $M$ and $L, L \ge 5$ in a distributed nested array, $T_{\min} = \frac{(L-4)M(M+1)}{2} + 2$ auxiliary sensors can be added at either $x_1 = 2M\,(M+1) - 1$ or $x_2 = \left(\frac{L}{2}+1\right) M\,(M+1) - 1$ to effectively fill the holes, thereby enabling unambiguous DOA estimation.

### *C. Subspace-based DOA estimation for distributed virtual linear apertures*

In Sec. III, $\mathbf{R}_{\bar{z}}$ is the covariance matrix of $\bar{\mathbf{z}}$. However, since $\bar{\mathbf{z}}$ represents only a single snapshot, covariance matrix has rank 1, which indicates that all sources are coherent. To achieve accurate DOA estimation, it is necessary to reconstruct a full-rank covariance matrix.

The spatial smoothing method partitions $\bar{\mathbf{z}}$ into several overlapping subarrays. Assuming that each subarray contains $K$ antennas, the total number of subarrays is given by $(L+1)M(M+1) - K + 1$. The corresponding array sample covariance matrix of each subarray is given by $\bar{\mathbf{z}}_i$ is

$$\mathbf{R}_i = \bar{\mathbf{z}}_i \bar{\mathbf{z}}_i^H \tag{34}$$

The reconstructed sample covariance matrix for the entire array is then obtained by averaging the sample covariance matrices of all subarrays

$$\mathbf{R}_{new} = \frac{1}{Q}\sum_{i=1}^{Q} \mathbf{R}_i \tag{35}$$

where $Q = (L+1)M(M+1) - K + 1$. The reconstructed matrix $\mathbf{R}_{new}$ successfully restores the full-rank structure, enabling reliable DOA estimation through standard subspace-based algorithms (MUSIC or ESPRIT) [1].

## V. Performance analysis

We now assess the performence of the proposed method. It is important to note that the effectiveness here refers to the spectrum of MUSIC.

As show in Fig. 2, We selected $M = 2$, $L = 5$, and by calculation, we can get $T = 5$, and select the left insertion position $x_1 = 11$. Among them, (a) is $\mathbb{S}_{\mathrm{DisNA}} \cup \mathbb{S}_{\mathrm{Aux}}$, (b) is $\mathbb{DIFF}_{\mathbb{S}_{\mathrm{DisNA}}+}$, (c) is $\mathbb{DIFF}_{(\mathbb{S}_{\mathrm{NA1}},\mathbb{S}_{\mathrm{Aux}})+}$, (d) is $\mathbb{DIFF}_{(\mathbb{S}_{\mathrm{Aux}},\mathbb{S}_{\mathrm{NA2}})+}$, (e) is $\mathbb{DIFF}_{(\mathbb{S}_{\mathrm{DisNA}},\mathbb{S}_{\mathrm{Aux}})+}$.

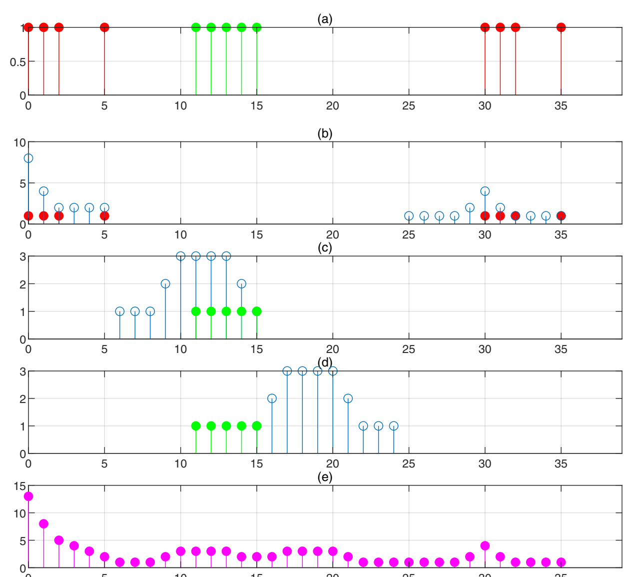


Fig. 2. Arrays structure

### A. Effectiveness

MUSIC is used to verify the effectiveness of the proposed, SNR = 0 dB, angle is $[-50^\circ : 25^\circ : 50^\circ]$

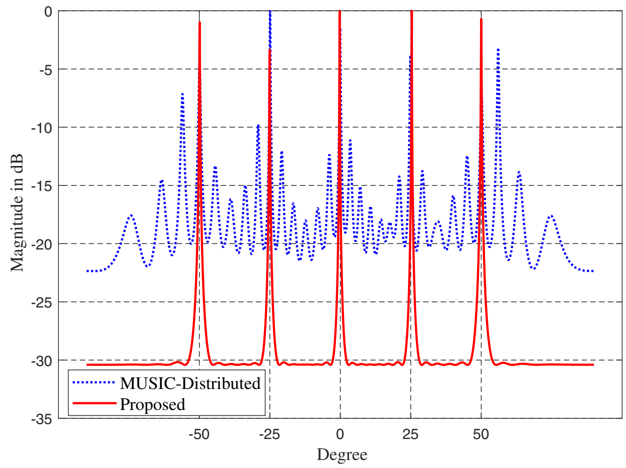


Fig. 3. MUSIC spectrum for DOA estimation.

Fig. 3 shows that when SNR = 0 dB, $\mathbb{S}_{\mathrm{DisNA}}$ directly performs MUSIC, which will produce a large number of pseudo peaks. The proposed method can construct a hole-free virtual ULA, eliminating the holes and achieving high-precision DOA angle estimation.

### B. Estimation performance

The SNR varies in the range of $[-8\mathrm{dB} : 2\mathrm{dB} : 10\mathrm{dB}]$. The angle is $[-50^\circ : 25^\circ : 50^\circ]$. The proposed array uses MUSIC and ESPRIT algorithms for DOA estimation. We compare its estimation performance with the traditional dual-scale ESPRIT (DS-ESPRIT) algorithm [14], [15] and the ULA of the same aperture size using the MUSIC algorithm.

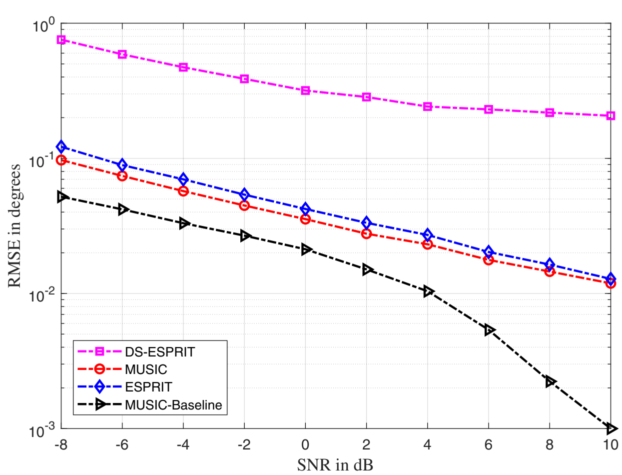


Fig. 4. RMSE versus SNR.

Fig. 4 shows that the proposed method, has better performance than the traditional DS-ESPRIT algorithm when using MUSIC and ESPRIT, but slightly worse than those of the ULA that has the same aperture size. However, the algorithm proposed in this paper achieves an estimation performance close to that of ULA with the same aperture size by using a smaller number of array elements, so reducing hardware cost and complexity.

## VI. Conclusion

This paper proposed an unambiguous hole-filling method for large-baseline distributed nested arrays by strategically inserting a continuous auxiliary array. We derived closed-form expressions for the minimum number of auxiliary elements and their optimal placement to ensure a hole-free difference coarray. This reconstruction enables high-resolution, ambiguity-free DOA estimation. Numerical simulations demonstrated that our approach effectively eliminates angular ambiguities and achieves DOA estimation accuracy comparable to a ULA with an equivalent aperture.

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