Wind lidar axes optimization for gust reconstruction

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***Abstract*—To reduce the environmental impact of civil aviation, future aircraft configurations will be equipped with Very High Aspect Ratio Wings. Such wings enable to reduce the drag but its structural sizing is a challenge to minimize the wing weight due to high structural constraints. A technique consists in using adaptive wings that modify their shape according to the gust measured ahead of the airplane using wind lidar. To determine the 3D wind field, the lidar is addressed along different axes to obtain the projections of the wind along each of them. In this paper, we present a methodology for optimizing lidar axis angles to best estimate the 3D wind field.**

***Index Terms*—min-max optimization, robust optimization, wind lidar, gust load alleviation, 3D wind field, turbulence.**

I. Introduction

Nowadays, Very High Aspect Ratio Wing is a promising technology enabling to reduce airplane drag and thus the aircraft environmental impact. However, for these geometries, the structural sizing is subject to many loads including the ones caused by turbulence. Thus, characterization of gust using wind lidar is important to perform gust load alleviation (GLA). The latter consist in actively reducing the loads caused by gust on aircraft's wings using actuators that modify the aerodynamic profile of the aircraft according to the direction and strength of the wind encountered. The use of a lidar to detect the wind structure in advance (so called feed-forward GLA) give enough time to the actuators to adapt the wings. This requires to measure the variation of the 3-dimensional (3D) wind velocity along the plane path ahead of the airplane. The wind measurement is typically performed with a direct detection molecular lidar [1] located at the front of the airplane. However, the instrument only measures the projection of the wind along its axis. To determine the 3D wind field, the lidar axis is typically addressed along multiple angles to measure the wind vector along different lidar axes direction.

A method, based on wind lidar measurements, for estimating and detecting a gust has been developed [2]. By accumulating the measurements over time, a nonlinear filter is used to estimate the gust where lidar axes are evenly distributed on a cone of arbitrary angle.

In a more general framework, we propose to optimize the lidar configuration with respect to the axes angles. An optimization method to minimize the gust estimation error is developed when the gust is immersed in Von-Karman-type turbulence [3]. The optimization criterion is based on the Cramér-Rao lower bound which depends on the unknown gust. To overcome this problem, we propose a min-max approach where the goal is to minimize the estimation error when the gusts belong to a set of critical gusts to which the aircraft is sensitive. To our knowledge, no previous studies have addressed this issue.

We begin by describing the parametric modeling of the gust and lidar measurements in section II. In section III, a min-max optimization method by generating a sample of critical gusts is developed. Section IV is devoted to simulation results demonstrating the robustness of the proposed optimization method.

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II. Problem modeling

In this section, we describe parametric models of gust and lidar wind measurements. The gust is modeled in 3-dimensional space, characterized by a 6-dimensional vector.

The unit vector $\mathbf{e}$ gives the direction of the wind in the gust, while the gust propagation direction is given by the unit vector $\mathbf{u}$ orthogonal to $\mathbf{e}$: $\mathbf{u}(\alpha, \beta) \perp \mathbf{e}(\alpha, \beta, \gamma)$ (Fig. 1, 2).

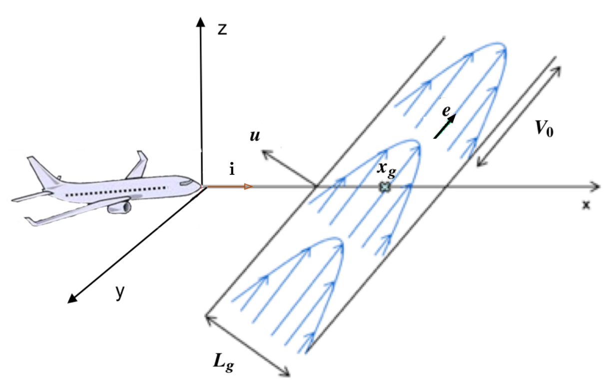

Fig. 1: Gust illustration.

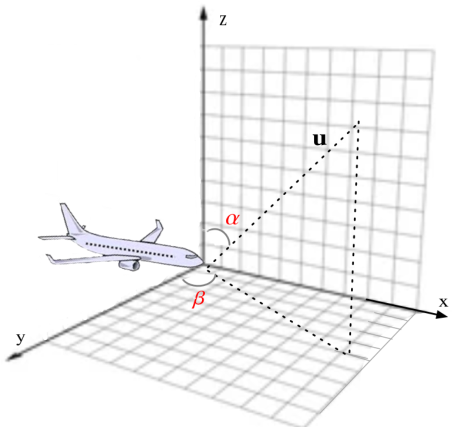

Fig. 2: Gust propagation direction.

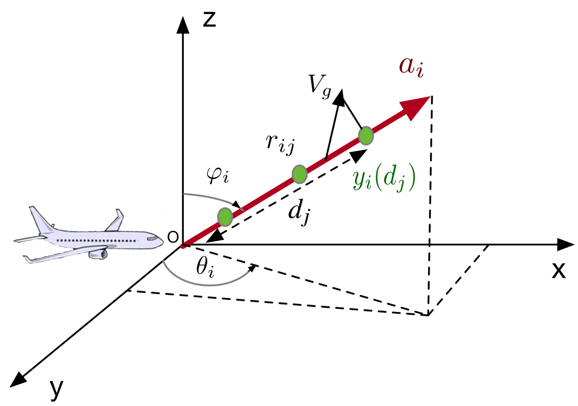

Fig. 3: Lidar axes and Doppler measurements.

These vectors are expressed as follows:

$$\begin{cases} \mathbf{u} = (\sin\alpha\,\sin\beta, \sin\alpha\,\cos\beta, \cos\alpha)^T \\ \mathbf{e} = (\cos\alpha\,\sin\beta\,\sin\gamma + \cos\beta\,\cos\gamma, \cos\alpha\,\cos\beta\sin\gamma \\ \quad - \sin\beta\,\cos\gamma, -\sin\alpha\,\sin\gamma)^T. \end{cases} \tag{1}$$

We assume that the gust has a typical '$1 - \cos$' shape [4]. The velocity vector field at point $\mathbf{r}$ is the following:

$$V_g(\mathbf{r}) = \begin{cases} \frac{V_0}{2}\left(1 + \cos\left(\frac{2\pi}{L_g}s\right)\right)\mathbf{e} & \text{if } -\frac{L_g}{2} \leq s \leq \frac{L_g}{2} \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $s = \mathbf{u} \cdot (\mathbf{r} - x_g\,\mathbf{i})$. The location of the maximum wind amplitude on the airplane axis is denoted by x_g. L_g is the gust length. The 3D gust model is thus determined by the following 6-dimensional state vector we aim to estimate with the lidar measurements:

$$X = (V_0, L_g, x_g, \alpha, \beta, \gamma)^T. \tag{3}$$

A. lidar measurements

The wind Doppler lidar is a useful equipment for measuring the wind at a given range. It consists in a laser emission sent to the atmosphere thanks to a telescope, and a scanner if lidar beam steering is needed. The molecules and aerosols of the atmosphere scatter the laser emission back to the receiver. During the scattering process, the Doppler effect shifts the laser frequency by $-2v_r/c$, where v_r is the radial component of the wind on the lidar axis (positive if the wind is moving away from the lidar, negative otherwise). The Doppler frequency shift is then analyzed with an optical interferometer and its dedicated signal processing. We consider N lidar axes at the front of the aircraft. Doppler measurements on each axis are perturbed by Von a Karman-type turbulence [3] whose spatial correlation properties are assumed to be known.

The distances from the lidar to the measurement points are denoted by d_j for $j = 1, \cdots, K$. The Doppler measurements $y_i(d_j)$ are the projection of the wind field on the lidar axes a_i (Fig. 3). According to (2), measurements on the axis $\mathbf{a}_i$ are expressed as follows:

$$y_i(d_j) = F_{ij}(\mathbf{u}, L_g, x_g)\; V \cdot \mathbf{a}_i + \epsilon_i(d_j) \stackrel{\text{def}}{=} h_{ij}(X) + \epsilon_i(d_j), \tag{4}$$

where,

$$\begin{cases} F_{ij} = 1 + \cos\left(\frac{2\pi}{L_g}\mathbf{u} \cdot (d_j\,\mathbf{a}_i - x_g\,\mathbf{i})\right) \\ V = \frac{V_0}{2}\,\mathbf{e}, \end{cases} \tag{5}$$

for $i = 1, \cdots, N$ and $j = 1, \cdots, K$. The lidar measurements are affected by a noise $\epsilon_i(d_j)$ due to the turbulence which is assumed to be isotropic an homogeneous. The intensity of the turbulence is denoted by σ_T. The noises $\epsilon_i(d_j)$ are assumed to be Gaussian with mean 0 and variance σ_T^2. The correlation between $\epsilon_i(d_j)$ at the point r_{ij} and $\epsilon_k(d_l)$ at point r_{kl} depends on the relative position of these points (Fig. 3) in accordance with the Von Karman model [3]. The following vector $\mathbf{y}$ of dimension $NK \times 1$ stacks the measurements made on all lidar axes:

$$\mathbf{y} = \mathbf{F}(L_g, x_g, \mathbf{u})\,\mathbf{a}^T\,V + \epsilon \stackrel{\text{def}}{=} h(X) + \epsilon, \tag{6}$$

where the components of the matrix $\mathbf{F}$ are F_{ij} for $i = 1, \cdots, N$ and $j = 1, \cdots, K$ and where the matrix $\mathbf{a}$, of dimension $N \times 3$, is composed of the N lidar axes.
Measurements $\mathbf{y}$ are affected by turbulence modeled as zero mean Gaussian noise ϵ with covariance matrix the Von Karman's correlation matrix Σ. This matrix is proportional to σ_T^2, where σ_T is the turbulence intensity, and depends on L_0, the large turbulence scale. In the following, we assume that L_0 and σ_T are known. The Von Karman's correlation matrix of dimension $(NK \times NK)$ is described in [3].

B. Measurement model

Measurements are accumulated over time, as the airplane moves forward. Since the speed of the airplane is high compared with that of the gust, we assume that the gust shape is constant during the sampling period ($< 0.5\,\mathrm{s}$). We then consider the following measurement gust model at time k,

$$\mathbf{y}_k = h_k(X) + \epsilon_k, \tag{7}$$

where ϵ_k and h_k are defined in (6). The gust state vector X (3) is considered to be constant. We assume that the turbulence noises are time to time decorrelated. The measurement model $h_k(X)$ depends on time k because it takes into account the aircraft's displacement of the airplane along its axis between times $k-1$ and k. This displacement is supposed to be known exactly. A nonlinear filtering is presented in [2] which estimates the gust state vector X at each time k. Our aim is to optimize the lidar axes configuration in order to minimize the estimation error of X at the last moment n.

III. Optimization of lidar axes configuration

A. Sequential Cramér-Rao Lower Bound

We consider N lidar axes $\mathbf{a}_i$ defined by the following angles (Fig. 3):

$$\Theta = \{\theta_i, \varphi_i\}_{1 \leq i \leq N}. \tag{8}$$

We seek to minimize the gust error estimation at the last moment (n) of measurement, i.e we want to find the best lidar axes angles that achieve this minimization. For this purpose, we use the Cramér-Rao Lower Bound [5]. The Cramér Lower Bound (CRLB) gives the minimal covariance matrix of any unbiased estimator $\hat{X}$.

$$\mathbb{V}(\hat{X}) \succeq CRLB(X) = J(X)^{-1} \tag{9}$$

in the sense that $\mathbb{V}(\hat{X}) - CRLB(X)$ is semi-definite positive. The square roots of the diagonal of CRLB represent the minimal std dev of any unbiased estimator of X (3). $J(X)$ is the information matrix. Based on the measurement model (6), J_n at the last measurement's time n can be calculated recursively as follows [5]:

$$\begin{aligned} J_n(\Theta, X) = &\left(\frac{\partial h_n}{\partial X}\right) \Sigma^{-1} \left(\frac{\partial h_n}{\partial X}\right)^T \\ &+ \Phi^{-1}(n-1,n)^T J_{n-1}(\Theta, X)\, \Phi^{-1}(n-1,n), \end{aligned} \tag{10}$$

with $J_0 = P_0^{-1}$ where P_0 is the covariance matrix of the a priori distribution of X, reflecting the initial uncertainty, where Φ is the transition matrix. Since the gust is assumed to be constant during the aircraft's movement, $\Phi(n-1,n)$ is the identity matrix. Note that the derivative $\left(\frac{\partial h_n}{\partial X}\right)$ and the Von Karman correlation matrix Σ depend on lidar axes angles Θ (8). Indeed, Σ depends on the spatial correlations between the measurements points r_{ij} on the lidar axes (Fig. 3).

B. Optimization criterion

The optimization criterion will be based on the CRLB at the last measurement's time. Specifically, we'll use the following criterion:

$$\begin{aligned} \sigma_B(\Theta, X) &= -\log\det\left(J_n(\Theta, X)\right) \\ &= \log\det\left(CRLB_n(\Theta, X)\right), \end{aligned} \tag{11}$$

where J_n is defined in (10). We want to minimize this criterion with respect to the lidar angles Θ. This criterion represents global information about the state X given the measurements. At measurement time k to minimize this criterion using gradient descent algorithms, we must compute the following derivatives:

$$\frac{\partial}{\partial \Theta_i} \log\det(J_k) = \mathrm{Tr}\left(CRLB_k(\Theta) \frac{\partial}{\partial \Theta_i} J_k(\Theta)\right), \tag{12}$$

which involves the following derivatives (10):

$$\begin{aligned} \frac{\partial}{\partial \Theta_i} H_k = &\left(\frac{\partial h_k}{\partial X}\right)^T \frac{\partial \Sigma^{-1}}{\partial \Theta_i} \left(\frac{\partial h_k}{\partial X}\right) \\ &+ \frac{\partial}{\partial \Theta_i} \left(\frac{\partial h_k}{\partial X}\right)^T \Sigma^{-1} \left(\frac{\partial h_k}{\partial X}\right) \\ &+ \left(\frac{\partial h_k}{\partial X}\right)^T \Sigma^{-1} \frac{\partial}{\partial \Theta_i} \left(\frac{\partial h_k}{\partial X}\right), \end{aligned} \tag{13}$$

where:

$$\begin{cases} H_k \stackrel{\text{def}}{=} \left(\frac{\partial h_k}{\partial X}\right) \Sigma^{-1} \left(\frac{\partial h_k}{\partial X}\right)^T \\ \frac{\partial \Sigma^{-1}}{\partial \Theta_i} = -\Sigma^{-1} \frac{\partial \Sigma}{\partial \Theta_i} \Sigma^{-1}. \end{cases} \tag{14}$$

Derivatives at the last measurement time $\frac{\partial}{\partial \Theta_i} J_n(\Theta)$ are calculated recursively by means of (10). All of derivatives that appear in (14) can be calculated explicitly using the measurement equation (6).

C. Min-max optimization problem

The information matrix J_n (10) depend on the unknown gust state vector X (3), so the minimum of $\sigma_B(\Theta, X)$ with respect to Θ (11) varies with X. We suggest the following approach to overcome this problem. The airplane with larger wing aspect ratio is very sensitive to gusts perpendicular to the plane formed by the wings, for which the vector $\mathbf{e}$ (1) is perpendicular to the plane xy and for which the vector $\mathbf{u}$ is parallel to the Ox axis (Fig. 1). The set of such gusts considered critical is denoted by C. The optimization will focus on critical gusts. We want to find the lidar axes angles Θ that minimize the largest estimation error when the gusts belong to C (the worst-case value). The optimization we are considering is thus treated as a min-max problem described as follows:

$$\min_{\Theta} \max_{X \in C} \sigma_B(\Theta, X). \tag{15}$$

There is no guarantee of a global saddle point for this problem, since the criterion $\sigma_B(\Theta, X)$ is a priori neither convex in Θ nor concave in X. Classical Newton min-max algorithms [6] cannot be applied. We propose the following simplified optimization problem:

$$\min_{\Theta} \max_{X_1, \cdots, X_m \in C} \sigma_B(\Theta, X_1, \cdots, X_m). \tag{16}$$

The sample $\{X_i\}_{1 \leq i \leq m}$ is generated according to a Gaussian distribution in which the vector $\mathbf{e}$ (Fig. 1) is sampled around critical values perpendicular to the plane xy. We assume that,

for a sufficiently large m, both optimizations (15) and (16) are roughly equivalent.
In this formulation (16), the optimization can be solved by the routine fminimax from Matlab using a Sequential Quadratic Programming (SQP) method [7]. Using derivatives (12), (14) convergence is fast. However, there is no guarantee that a global minimum will be reached. Several random initializations of the parameter Θ are required.

IV. Numerical Simulations

We restrict ourselves to $N = 4$ axes with axial symmetry around the axis Ox (Fig. 3). The aim of future work will be to remove this constraint. Minimization of the criterion $\sigma_B(\Theta, X)$ (16) focuses on 4 lidar angles:

$$\Theta = (\theta_1, \phi_1, \theta_2, \phi_2)^T. \tag{17}$$

Due to axial symmetry, the 4 lidar axes whose configuration we want to optimize with respect to Θ are expressed in the following way:

$$\begin{cases} \mathbf{a}_1 = (\sin(\varphi_1)\sin(\theta_1), \sin(\varphi_1)\cos(\theta_1), \cos(\varphi_1))^T \\ \mathbf{a}_2 = (\sin(\varphi_2)\sin(\theta_2), \sin(\varphi_2)\cos(\theta_1), \cos(\varphi_2))^T \\ \mathbf{a}_3 = (\sin(\varphi_1)\sin(\theta_1), -\sin(\varphi_1)\cos(\theta_1), \cos(\varphi_1))^T \\ \mathbf{a}_4 = (\sin(\varphi_2)\sin(\theta_2), -\sin(\varphi_2)\cos(\theta_2), \cos(\varphi_2))^T \end{cases} \tag{18}$$

A. Scenario data

We consider a scenario with the following data:

- Means of the gust parameters: $\bar{V}_0$ = 10m/s, $\bar{L}_g$ = 200 m, $\bar{x}_g$ = 200 m, $\bar{\alpha}$ = 90 deg, $\bar{\beta}$ = 90 deg, $\bar{\gamma}$ = ± 90 deg
- Measurements: 21 evenly measurement points distributed over the interval [100m, 300m] for each lidar axis, measurements are taken every 0.05 s for 0.6s
- Large turbulence scale $L_0 = 750$ m, turbulence standard deviation: $\sigma_T = 2$ m/s
- Aircraft speed = 200 m/s

The critical gust samples $\{X_i\}_{1 \le i \le m}$ (16) are generated according to the following mixture of two Gaussian distributions,

$$X_i \sim \frac{1}{2}\phi(\bar{X}_1, \Omega) + \frac{1}{2}\phi(\bar{X}_2, \Omega), \tag{19}$$

where:

$$\begin{cases} \bar{X}_i = (\bar{V}_0, \bar{L}_g, \bar{x}_g, \bar{\alpha}, \bar{\beta}, \bar{\gamma}_i)^T \\ \Omega = \mathrm{diag}\,(\sigma^2_{V_0}, \sigma^2_{L_g}, \sigma^2_{x_g}, \sigma^2_{\alpha}, \sigma^2_{\beta}, \sigma^2_{\gamma}). \end{cases} \tag{20}$$

The distribution of the samples $\{X_i\}_{1 \le i \le m}$ is bimodal, with half of the samples generated at $\bar{\gamma}_1$ = 90 deg and the other half at $\bar{\gamma}_2$ = -90 deg. Variance values are set as follows: σ_{V_0} = 3m/s, σ_{L_g} = 50m, σ_{x_g} = 50m, σ_α = 20 deg, σ_β = 20 deg, σ_γ = 20 deg. The choice of σ_γ is such that unit vector $\mathbf{e}$, giving the direction of the wind in the gust, is close to ± 90 deg. At these values, the samples $\{X_i\}_{1 \le i \le m}$ belong to the critical gust region C (16) for the aircraft.

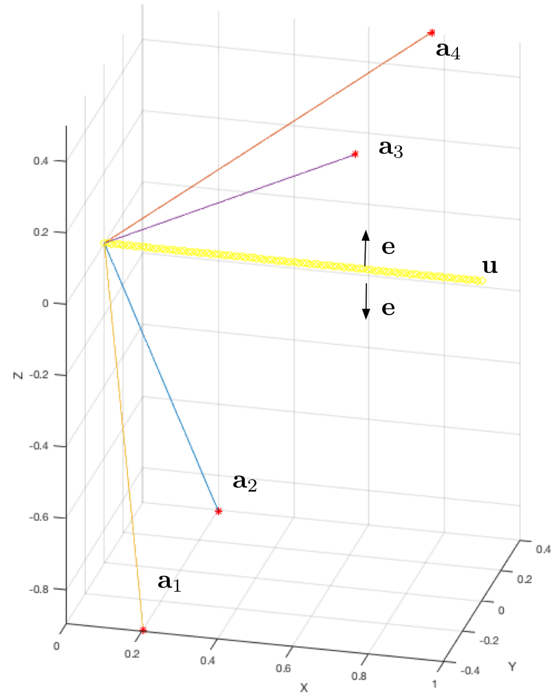

Fig. 4: Optimum lidar axes configuration.

B. Min-max optimization results

We consider $m = 100$ objective functions, i.e, we generate 100 critical gusts according to (19). For each value of the lidar angles Θ, the optimization algorithm fminimax from Matlab must calculate 100 evaluations of the criterion σ_B (11) and 100 evaluations of the gradient (12). Minimizing this criterion yields several local minima. Therefore, we have performed 50 initializations of Θ for the optimization algorithm.
The optimal lidar angles $\hat{\Theta}$ (17), expressed in degrees, are as follows: $\hat{\theta}_1 = 62.2$, $\hat{\phi}_1 = 115.9$, $\hat{\theta}_2 = 12.7$, $\hat{\phi}_2 = 22.7$. The angle between the lidar axis $\mathbf{a_1}$ and the line Ox is 36.6 deg and the angle between the lidar axis $\mathbf{a_2}$ and the line Ox is 85.1 deg. Fig. 4 shows the optimum lidar configuration.
To quantify the robustness of the min-max optimization, we compare the estimation error results obtained by a specific lidar configuration with those given by the min-max solution. The specific lidar configuration is obtained by minimizing the criterion $\sigma_B(\Theta, X)$ with respect to Θ by setting $X = \bar{X}$, which is the mean of the critical samples (20). The figures (Fig. 5 to Fig. 8) show the estimation errors (standard deviations) of the first 4 gust parameters components (3). The other components are not shown due to lack of space. For example, for the velocity V_0 (Fig. 5), we can see that the specific lidar configuration produces large errors for a large number of critical gusts. In contrast, the errors produced by the min-max solution are controlled over the entire range of critical gusts.

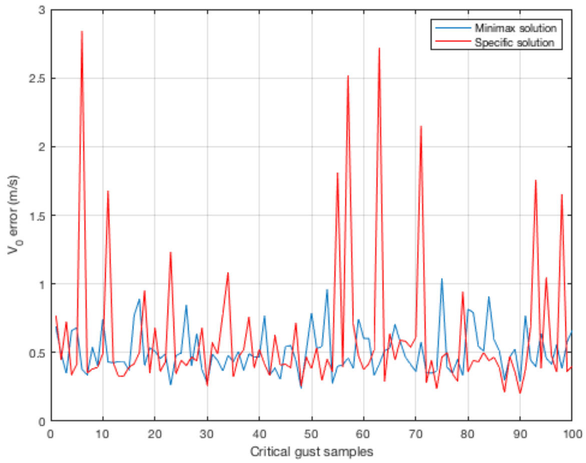

Fig. 5: Std dev of V_0 (m/s) for the min-max solution (blue) and specific solutions (red) as function of the critical samples.

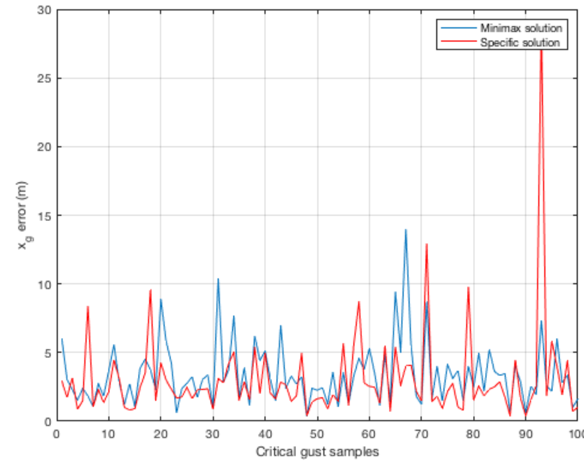

Fig. 7: Std dev of x_g (m) for the min-max solution (blue) and specific solutions (red) as function of the critical samples.

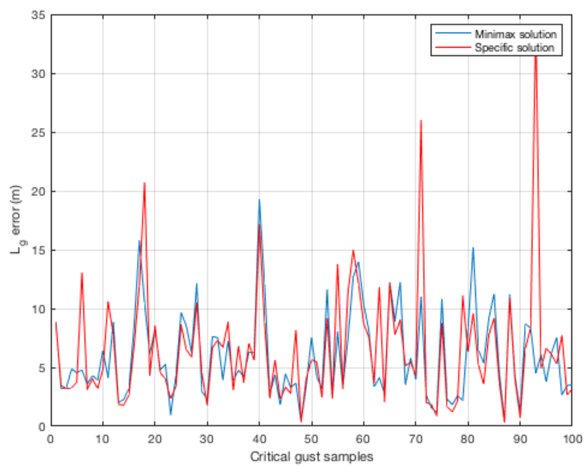

Fig. 6: Std dev of L_g (m) for the min-max solution (blue) and specific solutions (red) as function of the critical samples.

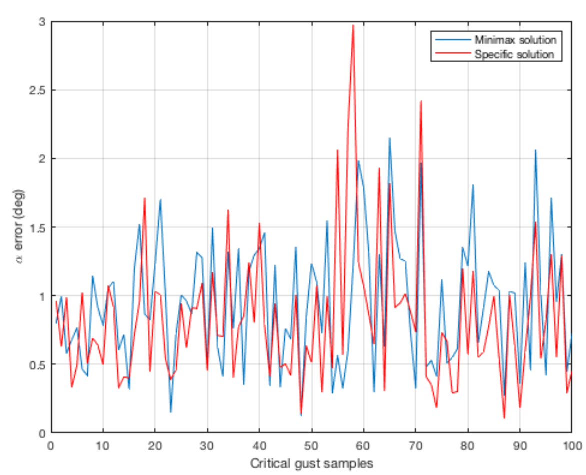

Fig. 8: Std dev of α (deg) for the min-max solution (blue) and specific solutions (red) as function of the critical samples.

V. Conclusion

We have proposed a method for optimizing the configuration of the wind lidar axes to best estimate the shape and the strength of gusts in turbulence. The optimization, based on the Cramér-Rao lower bound, is robust to gusts since it relies on min-max criterion in which the lidar axis angles are optimized for the set of gusts critical to the aircraft. To address local minima, more effective optimization methods can be considered [8]. Simulation results demonstrate the robustness of the proposed method. It would be interesting to estimate the parameters, assumed here to be known, that define the spatial correlations of the turbulence.

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