A Radar Pulse Compression Approach Based on Spline Interpolation

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***Abstract*—Pulse compression is employed in radar applications when it is necessary to jointly achieve sufficient sensitivity and good range resolution while employing low peak transmit power. Nonlinear frequency modulated waveforms are often considered to pursue these objectives. Pulse compression performances are typically measured based on the extension of the mainlobe and on the level of the sidelobes of the pulse autocorrelation function if a matched filter is used in reception. If not, the same criterion is used, applied to the output of the receive filter that is not matched to the pulsed waveform used in transmission. Several design methods have been proposed in the literature differing in the parametrization of the pulse, the optimization methods, and the requirements derived from the applications. In this paper, we propose a novel method of pulse compression based on the expression of the instantaneous frequency by means of cubic spline interpolation and on the optimization of an objective function related to the sidelobes energy. Several waveform design examples are shown, demonstrating how the proposed method allows extremely low sidelobes to be achieved, in comparison also to existing algorithms.**

***Index Terms*—Pulse compression, nonlinear frequency modulated waveform, spline interpolation, matched filter.**

I. Introduction

In several radar applications, the peak power required for the transmit pulse to achieve the desired range resolution, along with the total energy needed for the desired sensitivity or probability of detection of targets, may be excessive. In such cases, pulse compression is the solution. The basic principle of pulse compression is straightforward: a phase or frequency modulation is introduced in a constant amplitude transmit pulse with a given duration T, so that its autocorrelation function (ACF) changes from a triangular shape to one of the same duration (2T), but featuring a central narrow mainlobe and lower sidelobes. Amplitude tapering (windowing) can also be introduced in order to further reduce the level of the sidelobes, at the expense of some peak level reduction and broadening of the mainlobe [1].

This is true if one assumes that the receiver filter is matched to the waveform used in transmission. Using a mismatched filter at the receiver is an alternative [2] [3] [4]. In this case, the receiver output is given by the cross-correlation between the transmit waveform and the impulse response of the receiver, so that the overall system performance depends on the design of both the transmit waveform and receive filter response.

The requirements imposed on the design process depend on the application. Extremely low sidelobes of the receiver filter output are necessary when dealing with distributed targets featuring a normalized radar cross section highly variant with range. This can be encountered, for instance, in weather radars or in synthetic aperture radars.

A. Related work

Most studies on radar pulse compression have traditionally focused on the classical linear frequency modulated (LFM) rectangular pulse [1]. Early research by Fowle [5], as well as more recent studies (e.g., [6] [7] [8] [9]), have explored nonlinear frequency modulation (NLFM) pulses, leveraging the principle of stationary phase for their design. In [10], the stationary phase approach has been used in an iterative procedure aiming at minimizing the sidelobes energy (waveforms designed with that approach were used in [11] for the analysis and compensation of the effects of nonlinearities in power amplifiers).

The basic observation ruling a good pulse compression design is that the instantaneous frequency should deviate from linearity at the leading and trailing edges, where the highest frequency variations occur. This aspect was first noted by Cook and Paolillo [12] as beneficial for sidelobe suppression. This principle has been widely acknowledged in the literature and stated in several ways by using different parametric definitions of the pulse instantaneous frequency. For instance, a combination of linear and either tangent (LFM/tan-FM) or hyperbolic functions were used in [13] and [14], respectively; a tangent function was instead proposed in [15]. Piecewise polynomial functions to model the instantaneous frequency have been used in several studies [16] [17] [18]. Piecewise parabolic functions were used in [19], whereas the design was based on Beziers curves in [20].

A proper waveform parametrization along with the selection of effective solvers for the resulting objective function optimization are of paramount importance for the success of the design procedure. As to the objective function, its construction is often based on the waveform autocorrelation function, corresponding to the output of a matched filter receiver when the input is not affected by Doppler. As to the solvers, a variety of techniques have been chosen in the literature: for instance, genetic algorithms in [20] [8] and simulated annealing in [7].

In this work, we propose a new method to design a NLFM pulse based on the modeling of the instantaneous frequency through splines functions. The parameters to be optimized are the values of the instantaneous frequency at nonuniformly spaced knots and the function that is searched for is obtained by means of cubic spline interpolation. The position of the knots is fixed, so that the instantaneous frequency is a linear function of the free parameters. The proposed approach presents the following advantages: first, the "amount" of nonlinearity used to define the waveform is limited, and this facilitates the optimization process; second, the degree of regularity provided by the cubic spline interpolation is also the key to achieve high-performance solutions. Several examples of pulse compression waveform design are presented, showing that the proposed approach outperforms other existing algorithms used for a comparison.

The paper is organized as follows: in Section II, the proposed method is described; in Section III some examples of waveform design are shown and compared to existing techniques; in Section IV some concluding remarks are drawn.

II. Proposed method

In this section, the proposed method to design a pulse compression waveform is described. It is assumed that the radar system uses a matched filter receiver, so that its output is proportional to the autocorrelation function of the transmitted waveform. First the signal model is introduced.

Let $s(t)$ be the complex envelope of the transmitted waveform, where

$$s(t) = w(t)e^{j2\pi\phi(t)}, \tag{1}$$

for $-\frac{T}{2} < t < \frac{T}{2}$, where T is the pulse duration, $w(t)$ is the pulse shape, and $\phi(t)$ is the phase modulation. The latter can be expressed as

$$\phi(t) = 2\pi \int_{-\frac{T}{2}}^{t} f_i(\alpha)\, d\alpha, \tag{2}$$

where $f_i(t)$ is the pulse instantaneous frequency. The function $f_i(t)$ is actually at the basis of our approach to waveform design. In fact, the instantaneous frequency is derived from a few parameters by means of cubic spline interpolation.

Assume that $f_i(t)$ is an odd function, so that we need to design only the positive portion. Assume also that, in the positive interval, $f_i(t)$ ranges from 0 to $\frac{\Delta f}{2}$, where Δf is the total frequency sweep. Consider a set of $N+2$ fixed knots $\{t_0, t_1, \ldots, t_N, t_{N+1}\}$ in the time domain, with $t_0 = 0$ and $t_{N+1} = \frac{\Delta f}{2}$, whereas $t_n = \nu_n$, for $n = 1, 2, \ldots, N$, with ν_n a set of instantaneous frequencies to be determined. In our design procedure, $f_i(t)$ is the cubic spline interpolation of the points (t_n, ν_n), $n = 0, 1, \ldots, N, N+1$. It is well-known that, for all $t \in [0, \frac{T}{2}]$, $f_i(t)$ is a linear function of the values ν_n, and using (2) and (1), we can conclude that $s(t)$ is a nonlinear function of $\{\nu_1, \nu_2, \ldots, \nu_N\}$. A graphical sketch of a possible choice of the knots t_n, the values ν_n, and the resulting interpolating function $f_i(t)$ is shown in Fig. 1.

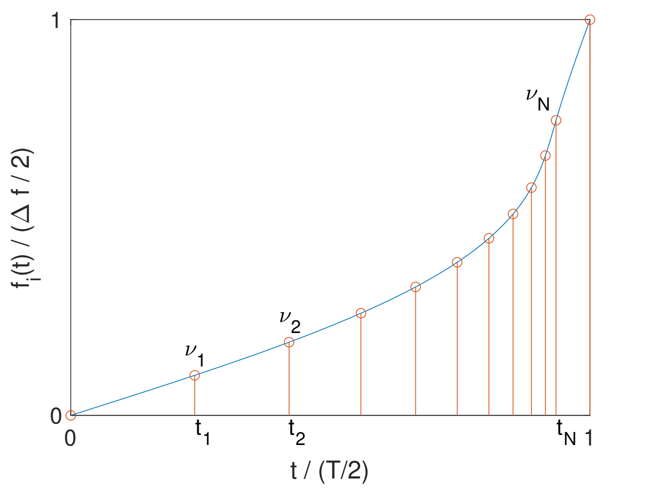

Fig. 1. A possible arrangement of the knots t_n, the respective values ν_n, and the interpolating function $f_i(t)$ (for the sake of simplicity, the variables are shown in a normalized scale).

The matched filter output is proportional to

$$R(\tau) = \int_{-\infty}^{\infty} s^*(t)s(t+\tau)\, dt. \tag{3}$$

The searched solution is a waveform $s(t)$ whose autocorrelation function $R(\tau)$ features a narrow mainlobe and low sidelobes. Assume that the mainlobe is located in the interval $\Omega_m = (-\sigma, \sigma)$ and that the sidelobes are located in $\Omega_s = (-\frac{T}{2}, -\sigma) \cup (\sigma, \frac{T}{2})$. The quantity σ is a value given as input to the design procedure.

The cost function we would like to minimize is the so called integrated sidelobe level (ISL), that is the energy of the sidelobes normalized to the total energy of the matched filter output, i.e.,

$$ISL = \frac{\int_{\Omega_s} |R(\tau)|^2\, d\tau}{\int_{-T}^{T} |R(\tau)|^2\, d\tau}. \tag{4}$$

Since $s(t)$ and, consequently, $R(\tau)$ and ISL, are nonlinear functions of the unknowns $\{\nu_1, \nu_2, \ldots, \nu_N\}$, nonlinear optimization methods must be used to achieve the final solution; therefore, an initial point must be given to the solver. In our procedure, the knots $\{t_1, t_2, \ldots, t_N\}$ have been selected as nonuniformly spaced between 0 and $\frac{T}{2}$, with a more dense concentration around $\frac{T}{2}$, where a higher nonlinear behavior of $f_i(t)$ is expected. To determine initial values for ν_n, a pulse compression waveform was designed by employing a standard technique (we used the classical stationary phase method proposed by Fowle [5]). After that, the instantaneous frequency was extracted and sampled in the instants t_n to achieve a first guess of ν_n. As to the solver used to achieve the optimal solution, the unconstrained minimization routine `fminunc` included in Optimization Toolbox by Matlab® was employed.

We would like to point out that the objective function ISL is related to the energy of the sidelobes, which is certainly a proper metric in the case of distributed targets. Alternatively, the peak-to-sidelobe level (PSL), defined as $PSL = 20\log_{10}\max_{\tau}|R(\tau)/R(0)|$, is also a commonly used indicator of the quality of a waveform design. The PSL and the ISL are usually aligned; in our experimental results we used both the ISL and the PSL in order to compare the outcomes of our proposed method and other design algorithms in the literature.

III. Test results

In this section, some examples of pulse compression waveform design obtained with the proposed method are shown and its performance is compared to that of some existing methods.

As already mentioned, some parameters need to be defined before running the proposed method. For example, the number of knots of the spline interpolation, as well as their positions, are not variables to be optimized, but are fixed. The null-to-null width of the mainlobe $\Omega_m = (-\sigma, \sigma)$ determines the range resolution of the system and strongly influences the performance of the design in terms of PSL and ISL; as in [10], such width is denoted as $\alpha(2/\Delta f)$, where α is a parameter and $2/\Delta f$ is the null-to-null mainlobe width relative to a linear frequency modulated (LFM) pulse with a total frequency sweep Δf (equal to the LFM pulse bandwidth if the product $\Delta f \cdot T$ is sufficiently high). The amplitude shaping window is another important feature of the designed waveform: in our design, we used a Tukey window, whose roll-off parameter r rules the width of the leading and trailing edges.

In Fig. 2, the PSL and the ISL of the ACF of the designed waveform are plotted vs. the number of knots of the spline interpolation. The knots are nonuniformly spaced by means of the warping law $1 - e^{-3t'}$, where t' denotes the time axis normalized with respect to $T/2$. For this example, the system parameters were: $T = 40$ μs; $\Delta f = 4.5$ MHz; $\alpha = 3.5$; $r = 0.1$. We found that in general a number of knots around $35 \div 40$ is optimal for the design.

In Fig. 3, the PSL and the ISL of the ACF are plotted vs. the range resolution of the designed waveform; note that the range resolution ΔR (expressed in meters) is obtained by multiplying 2σ (the null-to-null mainlobe width) by $c/2$, where c is the speed of light, i.e., $\Delta R = c \cdot \sigma$. For this example, the system parameters were: $T = 40$ μs; $\Delta f = 4.5$ MHz; $\alpha = 2.5 \div 6.0$; $r = 0.1$. As the range resolution degrades, it can be noted that the ISL shows a decreasing trend. The fluctuations around the trend can be attributed to the fact that the optimal solution depends not only on ΔR (i.e., α), but also on the setting of several other parameters, such as the number of knots, their positions, Δf and r. Therefore, by tuning such parameters for each specific resolution a continuous decrease is expected. In any case, based on Figs. 2 and 3, we can observe that, as expected and already mentioned, the ISL and the PSL indices exhibit similar trends with respect to different independent variables, so that minimizing the ISL corresponds very likely to optimizing also the PSL.

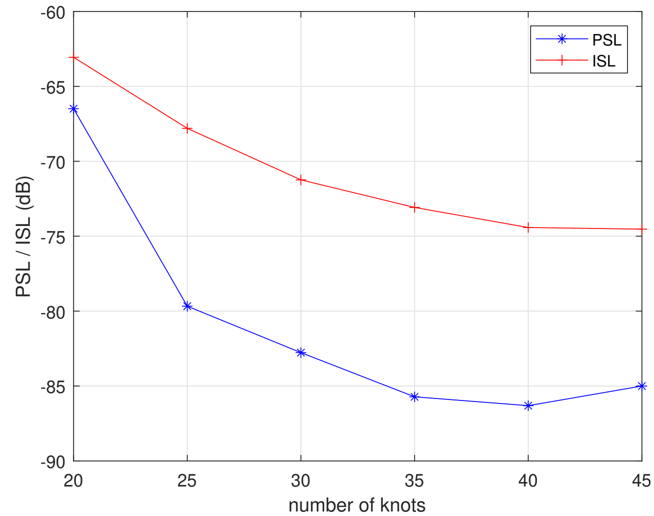

Fig. 2. PSL and ISL of the ACF vs. the number of knots of the spline interpolation. Waveform designed with system parameters $T = 40$ μs; $\Delta f = 4.5$ MHz; $\alpha = 3.5$; $r = 0.1$.

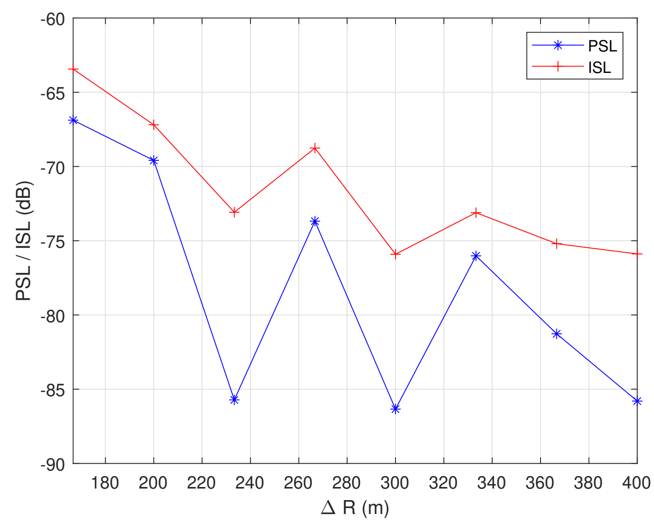

Fig. 3. PSL and ISL of the ACF vs. null-to-null range resolution ΔR. Waveform designed with system parameters $T = 40$ μs; $\Delta f = 4.5$ MHz; $r = 0.1$.

In Table I, some results obtained with the proposed method are compared with those of other techniques from the literature. More specifically, the table shows the results obtained with: the method proposed in [7], based on simulated annealing and denoted as SA, with system parameters $T = 40$ μs, $\Delta f = 5$ MHz, $r = 0.1$; the iterative stationary phase (ISP) method proposed in [10], with system parameters $T = 40$ μs, $\Delta f = 5$ MHz, $r = 0.15$, and with two different values of ΔR (coming from $\alpha = 4.0$ and $\alpha = 4.5$); the method proposed in this paper with system parameters $T = 40$ μs, $\Delta f = 4.5$ MHz, $r = 0.1$. Range resolution is expressed both in terms of

TABLE I
RESULTS OF PULSE COMPRESSION DESIGN FOR $T = 40$ μS.

Method	PSL (dB)	ISL (dB)	ΔR (m)	ΔR_{3dB} (m)
SA	-68.7	-	-	65
ISP-1	-74.5	-67.3	240.0	55.5
ISP-2	-78.9	-70.4	270.0	57.8
proposed	-85.7	-73.1	233.3	57.3

ΔR and of the -3 dB mainlobe width, denoted as ΔR_{3dB}. As can be seen, the proposed method surpasses the other ones in terms of any index (PSL, ISL and range resolution). The ACF of the pulse waveform obtained with the proposed method is depicted in Fig. 4.

In order to highlight the influence of the parameter r of the amplitude shaping window, we plot in Fig. 5 the ACF of the waveform designed with the same parameters as that in Fig. 4, apart from $r = 0.05$. A reduced rise time of the leading and trailing edges of the pulse more accurately represents the real situation in which the power amplifier operates in the saturation region most of the time. As shown, halving the rise time results in only a limited degradation of overall performance.

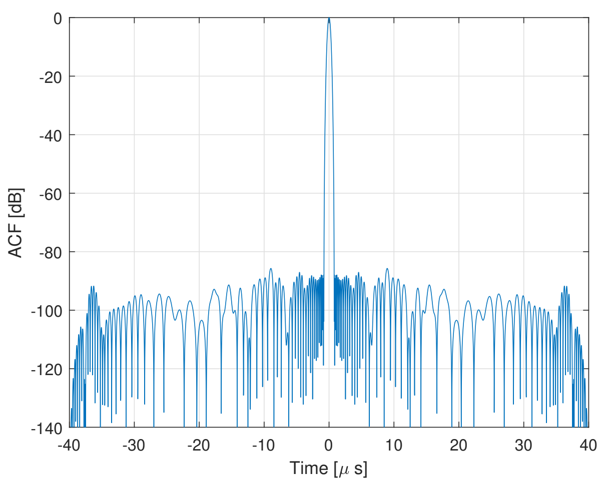

Fig. 4. ACF of the pulse designed with the proposed method and system parameters: $T = 40$ μs; $\Delta f = 4.5$ MHz; $\alpha = 3.5$, $r = 0.1$.

Analogous results for different system parameters are shown in Table II, which reports the results obtained with: the method proposed in [20], based on genetic algorithms and denoted as GA, with system parameters $T = 67$ μs; $\Delta f = 2.2$ MHz; the method proposed in [10], with system parameters $T = 67$ μs, $\Delta f = 2.2$ MHz, $r = 0.2$, and with two different resolutions, i.e., null-to-null mainlobe width; the proposed method with system parameters $T = 67$ μs, $\Delta f = 2.2$ MHz, $r = 0.1$. The ACF of the pulse waveform obtained with the proposed method is depicted in Fig. 6.

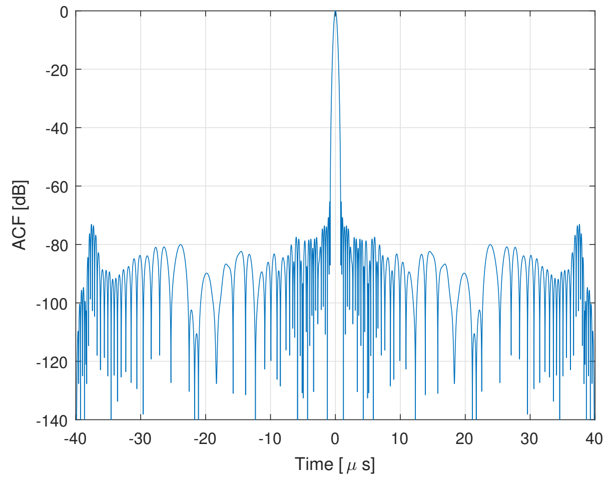

Fig. 5. ACF of the pulse designed with the proposed method and system parameters: $T = 40$ μs; $\Delta f = 4.5$ MHz; $\alpha = 3.5$, $r = 0.05$.

TABLE II
RESULTS OF PULSE COMPRESSION DESIGN FOR T=67 μS.

Method	PSL (dB)	ISL (dB)	ΔR (m)	ΔR_{3dB} (m)
GA	-59	-37	$\approx$ 400	120
ISP-1	-72.3	-65.3	477.3	118.9
ISP-2	-80.4	-69.2	613.6	130.6
proposed	-80.3	-69.2	477.3	119.2

IV. CONCLUSIONS

We have developed and presented a new method to design radar NLFM pulse waveforms exhibiting extremely low sidelobes in their autocorrelation function. Consequently, assuming that the radar receiver is matched to the transmitted pulse, such waveforms can be effectively employed in scenarios where a specific range resolution is required, but the use of

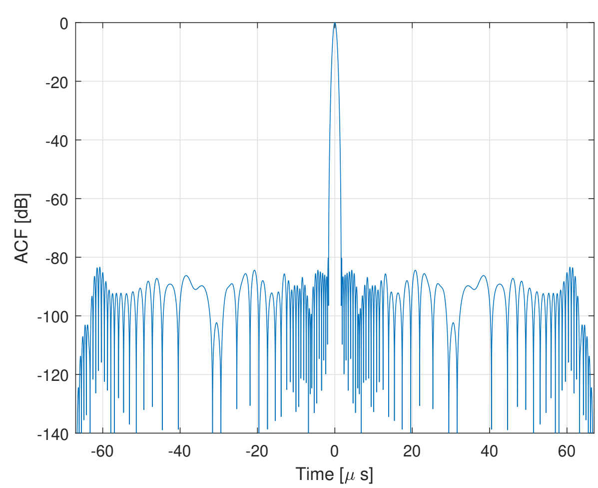

Fig. 6. ACF of the pulse designed with the proposed method and system parameters: $T = 67$ μs; $\Delta f = 2.2$ MHz; $\alpha = 3.5$, $r = 0.1$.

pulse compression is unavoidable due to limited peak power available in transmission, or there is a need of excellent compression performances to minimize mutual interference among targets, or both conditions occur.

Our new proposed method is based on a cubic spline model of the pulse instantaneous frequency, whose knots are fixed and nonuniformly spaced. The objective function is the ISL of the matched filter output (i.e., the ISL of the pulse ACF), and the effort is the optimization of the free parameters, which are the values of the instantaneous frequency in the knot positions. Due to the fixed position of the knots, the instantaneous frequency is a linear function of the free parameters, and this is relevant to speed up the optimization process. On the other hand, also the non-uniformity of the knots distribution is a key to achieve optimal solutions faster since it helps to control better the trailing edges of the instantaneous frequency while keeping the number of knots limited.

Obviously, the number of knots plays a crucial role in reducing the ISL; as we have shown for a specific example, an optimal number of knots ranges between 35 and 40, when combined with the warping law specified in the paper. Furthermore, it has been confirmed that minimizing the ISL consistently leads to lower levels of PSL. The method also offers very good flexibility in designing waveforms with different range resolutions while keeping both the frequency sweep and the amplitude taper factor unchanged.

We have compared the compression performance of the proposed design method with that obtained by applying other methods reported in the literature. The results obtained in this study have shown that the proposed method is able to achieve much better values of ISL and PSL, with an equal or even better range resolution.

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