An Efficient Two-Stage Weighted Least Squares Approach for Moving Target Localization in Distributed MIMO Radars

1st Abdollah Ajorloo
DIET Department
Sapienza University of Rome
Rome, Italy
abdollah.ajorloo@uniroma1.it

2nd Rouhollah Amiri
Department of Electrical Engineering
Sharif University of Technology
Tehran, Iran
amiri@sharif.edu

3rd Ali Noroozi
Department of Electrical Engineering
Sharif University of Technology
Tehran, Iran
ali_noroozi@sharif.edu

4th Fabiola Colone
DIET Department
Sapienza University of Rome
Rome, Italy
fabiola.colone@uniroma1.it

***Abstract*—In this paper, an algebraic closed-form solution for the moving target localization problem in a multiple-input multiple-output radar with widely separated antennas using bistatic range and bistatic range rate measurements is developed. In the first stage of the proposed method, a weighted least squares estimation is applied on a set of linear equations obtained via nuisance parameter elimination technique to estimate the position and velocity of the target. In the second stage, an estimate of the error in the solution of the first stage is obtained to refine and enhance the localization performance. The proposed algorithm is showcased theoretically and by simulations to achieve the Cramer-Rao lower bound performance under mild Gaussian noise conditions. Numerical simulations are included to evaluate performance of the proposed algorithm and verify the theoretical results.**

***Index Terms*—Moving target localization, multiple-input multiple-output (MIMO) radar, singular value decomposition (SVD), Bistatic range (BR), bistatic range rate (BRR).**

I. Introduction

With the rise of multiple-input multiple-output (MIMO) radar systems in recent years, numerous challenges have emerged, driving extensive research efforts [1]–[6]. In particular, target localization using distributed transmit and receive antennas has gained significant attention within the radar and communications communities over the past decade [7]–[10].

Distributed MIMO radars leverage spatial diversity to enhance detection and estimation accuracy, making them a powerful tool for target localization [11]. Various types of measurements, including bistatic range (BR), bistatic range rate (BRR), and angle of arrival (AOA), as well as their combinations, can be employed to estimate target parameters [8], [12]. The integration of these diverse measurements plays a crucial role in improving localization performance, particularly in challenging environments.

Several methods have been proposed to estimate target position using bistatic range (BR) measurements [13]–[17]. However, in moving target localization, both BR and BRR measurements must be utilized to accurately estimate both position and velocity. Furthermore, incorporating BRR measurements enhances localization accuracy by providing additional constraints on the target's motion.

In [18], the authors proposed an algorithm for estimating target position and velocity by dividing the measurements into multiple sets, each formed using a reference transmitter and all receivers (or vice versa). A closed-form two-stage weighted least squares (TSWLS) estimator was then applied separately to each set, and the individual estimates were combined to obtain the final result.

In [19], a different TSWLS approach was introduced, where, unlike [18], all measurements were processed together in the first stage. This method consolidates all nuisance parameters into a single reference parameter before applying two WLS estimators to derive the final solution.

In this paper, we propose an efficient closed-form algebraic TSWLS solution for moving target localization in distributed MIMO radar systems. In the first stage, we construct a set of linear equations from measurements by eliminating the nuisance parameters through a structured singular value decomposition (SVD) approach; these equations are then solved using a WLS estimator to obtain an initial estimate of the target's position and velocity. In the second stage, we refine this estimate by leveraging the inherent relationships between the nuisance parameters and the target's position and velocity, effectively compensating for estimation errors introduced in the first stage. Numerical simulations demonstrate that our approach outperforms existing localization algorithms in terms of accuracy and robustness, particularly in low and moderate noise scenarios.

II. Measurement Model

Consider a distributed MIMO radar system comprised of M transmitters and N receivers in a 3-D space. The location of the ith transmitter and the jth receiver are denoted by

$\mathbf{x}_{t,i} = [x_{t,i}, y_{t,i}, z_{t,i}]^T$ and $\mathbf{x}_{r,j} = [x_{r,j}, y_{r,j}, z_{r,j}]^T$, respectively, for $i = 1, \ldots, M$ and $j = 1, \ldots, N$. The velocity of the ith transmitter and the jth receiver are represented by $\dot{\mathbf{x}}_{t,i} = [\dot{x}_{t,i}, \dot{y}_{t,i}, \dot{z}_{t,i}]^T$ and $\dot{\mathbf{x}}_{r,j} = [\dot{x}_{r,j}, \dot{y}_{r,j}, \dot{z}_{r,j}]^T$ respectively, for $i = 1, \ldots, M$ and $j = 1, \ldots, N$. The position and the velocity of a desired target are denoted by $\mathbf{x}_0 = [x_0, y_0, z_0]^T$ and $\dot{\mathbf{x}}_0 = [\dot{x}_0, \dot{y}_0, \dot{z}_0]^T$, respectively.

The noise-free BR measurement for the (i, j) transmitter-receiver pair is defined as sum of transmitter-to-target range, $d_{t,i} = \|\mathbf{x}_0 - \mathbf{x}_{t,i}\|$, and target-to-receiver range, $d_{r,j} = \|\mathbf{x}_0 - \mathbf{x}_{r,j}\|$, and can be represented as

$$r_{i,j} = d_{t,i} + d_{r,j}, \tag{1}$$

The BR measurement in the presence of noise is modeled as $\hat{r}_{i,j} = r_{i,j} + \Delta r_{i,j}$, where $\Delta r_{i,j}$ is the measurement noise term. Collecting all BR measurements in a vector, yields

$$\hat{\mathbf{r}} = \mathbf{r} + \Delta\mathbf{r}, \tag{2}$$

where $\mathbf{r} = [\mathbf{r}_1^T, \ldots, \mathbf{r}_M^T]^T$, $\mathbf{r}_i = [r_{i,1}, \ldots, r_{i,N}]^T$, $\hat{\mathbf{r}} = [\hat{\mathbf{r}}_1^T, \ldots, \hat{\mathbf{r}}_M^T]^T$, $\hat{\mathbf{r}}_i = [\hat{r}_{i,1}, \ldots, \hat{r}_{i,N}]^T$, $\Delta\mathbf{r} = [\Delta\mathbf{r}_1^T, \ldots, \Delta\mathbf{r}_M^T]^T$, and $\Delta\mathbf{r}_i = [\Delta r_{i,1}, \ldots, \Delta r_{i,N}]^T$. The noise vector $\Delta\mathbf{r}$ is considered to be a zero-mean Gaussian random vector with covariance matrix

$$\mathbf{Q}_{\mathbf{r}} = \mathbb{E}\left[\Delta\mathbf{r}\Delta\mathbf{r}^T\right]. \tag{3}$$

The true BRR measurement for the (i, j)-th transmitter-receiver pair defined as the sum of transmitter-to-target range rate, $\dot{d}_{t,i} = \boldsymbol{\rho}_{\mathbf{x}_0,\mathbf{x}_{t,i}}^T(\dot{\mathbf{x}}_0 - \dot{\mathbf{x}}_{t,i})$, and target-to-receiver range rate, $\dot{d}_{r,j} = \boldsymbol{\rho}_{\mathbf{x}_0,\mathbf{x}_{r,j}}^T(\dot{\mathbf{x}}_0 - \dot{\mathbf{x}}_{r,j})$, is given by

$$\dot{r}_{i,j} = \dot{d}_{t,i} + \dot{d}_{r,j}. \tag{4}$$

Note that $\boldsymbol{\rho}_{\mathbf{a},\mathbf{b}} = (\mathbf{a} - \mathbf{b})/\|\mathbf{a} - \mathbf{b}\|$ denotes the unit vector directed from $\mathbf{b}$ to $\mathbf{a}$. In the presence of measurement noise, the BRR measurement can be modeled as $\hat{\dot{r}}_{i,j} = \dot{r}_{i,j} + \Delta\dot{r}_{i,j}$, where $\Delta\dot{r}_{i,j}$ is the noise term. The BRR measurements can be stacked in vector form as

$$\hat{\dot{\mathbf{r}}} = \dot{\mathbf{r}} + \Delta\dot{\mathbf{r}} \tag{5}$$

where $\dot{\mathbf{r}} = [\dot{\mathbf{r}}_1^T, \ldots, \dot{\mathbf{r}}_M^T]^T$, $\dot{\mathbf{r}}_i = [\dot{r}_{i,1}, \ldots, \dot{r}_{i,N}]^T$, $\hat{\dot{\mathbf{r}}} = [\hat{\dot{\mathbf{r}}}_1^T, \ldots, \hat{\dot{\mathbf{r}}}_M^T]^T$, $\hat{\dot{\mathbf{r}}}_i = [\hat{\dot{r}}_{i,1}, \ldots, \hat{\dot{r}}_{i,N}]^T$, $\Delta\dot{\mathbf{r}} = [\Delta\dot{\mathbf{r}}_1^T, \ldots, \Delta\dot{\mathbf{r}}_M^T]^T$, and $\Delta\dot{\mathbf{r}}_i = [\Delta\dot{r}_{i,1}, \ldots, \Delta\dot{r}_{i,N}]^T$. The BRR noise vector $\Delta\dot{\mathbf{r}}$ is also considered to be a zero-mean Gaussian vector with the covariance matrix $\mathbf{Q}_{\dot{\mathbf{r}}} = \mathbb{E}\left[\Delta\dot{\mathbf{r}}\Delta\dot{\mathbf{r}}^T\right]$.

In this paper, we aim to estimate the unknown vector $\mathbf{u} = \left[\mathbf{x}_0^T, \dot{\mathbf{x}}_0^T\right]^T$ using the measurement vector $\hat{\mathbf{m}} = \left[\hat{\mathbf{r}}^T, \hat{\dot{\mathbf{r}}}^T\right]^T = \mathbf{m} + \Delta\mathbf{m}$, where $\Delta\mathbf{m}$ is the total measurement noise vector with covariance matrix

$$\mathbf{Q}_m = \mathrm{blkdiag}\left(\mathbf{Q}_{\mathbf{r}}, \mathbf{Q}_{\dot{\mathbf{r}}}\right). \tag{6}$$

III. Closed-Form Solution

Stage 1: Rearranging (1) as $r_{i,j} - d_{t,i} = d_{r,j}$ and squaring both sides, after some algebraic manipulations, gives

$$\left(\mathbf{x}_{t,i}^T - \mathbf{x}_{r,j}^T\right)\mathbf{x}_0 = \frac{1}{2}\left(r_{i,j}^2 + \mathbf{x}_{t,i}^T\mathbf{x}_{t,i} - \mathbf{x}_{r,j}^T\mathbf{x}_{r,j}\right) - r_{i,j}d_{t,i}. \tag{7}$$

By taking the time derivative of (7), it follows that

$$\begin{aligned}&\left(\dot{\mathbf{x}}_{t,i} - \dot{\mathbf{x}}_{r,j}\right)^T\mathbf{x}_0 + \left(\mathbf{x}_{t,i} - \mathbf{x}_{r,j}\right)^T\dot{\mathbf{x}}_0 = \dot{r}_{i,j}r_{i,j}\\ &+\dot{\mathbf{x}}_{t,i}^T\mathbf{x}_{t,i} - \dot{\mathbf{x}}_{r,j}^T\mathbf{x}_{r,j} - \dot{r}_{i,j}d_{t,i} - r_{i,j}\dot{d}_{t,i}.\end{aligned} \tag{8}$$

Stacking (7) for the ith transmitter and all receivers yields, in matrix form,

$$\mathbf{S}_i\mathbf{x}_0 = \mathbf{z}_i + \mathbf{r}_i d_{t,i}, \tag{9}$$

where $\mathbf{S}_i(j,:) = \mathbf{x}_{t,i}^T - \mathbf{x}_{r,j}^T$ and $\mathbf{z}_i(j) = \frac{1}{2}\left(r_{i,j}^2 + \mathbf{x}_{t,i}^T\mathbf{x}_{t,i} - \mathbf{x}_{r,j}^T\mathbf{x}_{r,j}\right)$. Note that in (9), the target position $\mathbf{x}_0$ and the nuisance parameter $d_{t,i}$ are unknown and the nuisance parameter depends nonlinearly on the target position. To form a set of linear equations, it is essential to eliminate the nuisance parameter. To this end, we remove the nuisance parameter $d_{t,i}$ by premultiplying (9) by the matrix $\mathbf{M}_i$, of which the vector $\mathbf{r}_i$ is in the null space, which is denoted by

$$\mathbf{M}_i = \mathbf{V}^T\mathbf{D}_i, \tag{10}$$

where $\mathbf{D}_i = \left(\mathrm{diag}\left(\mathbf{r}_i\right)\right)^{-1}$ and the matrix $\mathbf{V}$ is obtained from the SVD of the matrix $(\mathbf{I}_N - \mathbf{Z})$ given by

$$(\mathbf{I}_N - \mathbf{Z}) = \left[\begin{array}{cc}\mathbf{U} & \mathbf{u}\end{array}\right]\left[\begin{array}{cc}\boldsymbol{\Sigma}_z & \mathbf{0}\\ \mathbf{0}^T & 0\end{array}\right]\left[\begin{array}{c}\mathbf{V}^T\\ \mathbf{v}^T\end{array}\right] = \mathbf{U}\boldsymbol{\Sigma}_z\mathbf{V}^T, \tag{11}$$

$$\mathbf{Z} = \left[\begin{array}{cc}\mathbf{0}_{N-1} & \mathbf{I}_{N-1}\\ 1 & \mathbf{0}_{N-1}^T\end{array}\right]_{N\times N} = \text{circular shift matrix}, \tag{12}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices of length $N \times (N-1)$ associated with nonzero singular values of $(\mathbf{I}_N - \mathbf{Z})$, $\mathbf{u}$ and $\mathbf{v}$ are vectors spanning the null space of $(\mathbf{I}_N - \mathbf{Z})$ and $(\mathbf{I}_N - \mathbf{Z})^T$, respectively, and $\boldsymbol{\Sigma}_z$ is a diagonal matrix with $N-1$ nonzero singular values of $(\mathbf{I}_N - \mathbf{Z})^T$. As $(\mathbf{I}_N - \mathbf{Z})\mathbf{D}_i\mathbf{r}_i = \mathbf{0}$, it follows that $\mathbf{V}^T\mathbf{D}_i\mathbf{r}_i$ is zero as well. Thus, we have

$$\mathbf{M}_i\mathbf{S}_i\mathbf{x}_0 = \mathbf{M}_i\mathbf{z}_i. \tag{13}$$

Taking the time derivative of (13) gives

$$\left(\dot{\mathbf{M}}_i\mathbf{S}_i + \mathbf{M}_i\dot{\mathbf{S}}_i\right)\mathbf{x}_0 + \mathbf{M}_i\mathbf{S}_i\dot{\mathbf{x}}_0 = \dot{\mathbf{M}}_i\mathbf{z}_i + \mathbf{M}_i\dot{\mathbf{z}}_i. \tag{14}$$

where $\dot{\mathbf{M}}_i = -\mathbf{V}^T\mathbf{D}_i^2\mathrm{diag}(\dot{\mathbf{r}}_i)$, $\dot{\mathbf{S}}_i(j,:) = \dot{\mathbf{x}}_{t,i}^T - \dot{\mathbf{x}}_{r,j}^T$ and $\dot{\mathbf{z}}_i(j) = r_{i,j}\dot{r}_{i,j} + \mathbf{x}_{t,i}^T\dot{\mathbf{x}}_{t,i} - \mathbf{x}_{r,j}^T\dot{\mathbf{x}}_{r,j}$. By stacking (13) and (14) separately for all transmitters and then combining them, it follows that

$$\mathbf{G}_1\boldsymbol{\theta} = \mathbf{h}_1, \tag{15}$$

where $\boldsymbol{\theta} = \left[\begin{array}{c}\mathbf{x}_0\\ \dot{\mathbf{x}}_0\end{array}\right]$, $\mathbf{h}_1 = \left[\begin{array}{c}\mathbf{h}_{1,1}\\ \mathbf{h}_{1,2}\end{array}\right]$, $\mathbf{G}_1 = \left[\begin{array}{c}\mathbf{G}_{1,1}, \mathbf{O}\\ \mathbf{G}_{1,2}, \mathbf{G}_{1,1}\end{array}\right]$,

$$\mathbf{h}_{1,1} = \left[\left(\mathbf{M}_1\mathbf{z}_1\right)^T, \ldots, \left(\mathbf{M}_M\mathbf{z}_M\right)^T\right]^T, \tag{16}$$

$$\mathbf{h}_{1,2} = \left[\left(\dot{\mathbf{M}}_1\mathbf{z}_1 + \mathbf{M}_1\dot{\mathbf{z}}_1\right)^T, \ldots, \left(\dot{\mathbf{M}}_M\mathbf{z}_M + \mathbf{M}_M\dot{\mathbf{z}}_M\right)^T\right]^T,$$

$$\mathbf{G}_{1,1} = \left[\left(\mathbf{M}_1\mathbf{S}_1\right)^T, \ldots, \left(\mathbf{M}_M\mathbf{S}_M\right)^T\right]^T,$$

$$\mathbf{G}_{1,2} = \left[\left(\dot{\mathbf{M}}_1\mathbf{S}_1 + \mathbf{M}_1\dot{\mathbf{S}}_1\right)^T, \ldots, \left(\dot{\mathbf{M}}_M\mathbf{S}_M + \mathbf{M}_M\dot{\mathbf{S}}_M\right)^T\right]^T.$$

We should consider the noise in the measured values. Replacing $\mathbf{r}$ and $\dot{\mathbf{r}}$ with $\hat{\mathbf{r}}-\Delta\mathbf{r}$ and $\hat{\dot{\mathbf{r}}}-\Delta\dot{\mathbf{r}}$ in (15) and ignoring the second-order noise terms gives the error vector as follows

$$\boldsymbol{\varepsilon}_1=\hat{\mathbf{h}}_1-\hat{\mathbf{G}}_1\boldsymbol{\theta}, \tag{17}$$

where $\boldsymbol{\varepsilon}_1=\mathbf{B}_1\Delta\mathbf{m}$, $\mathbf{B}_1=\begin{bmatrix}\mathbf{B}_{1,1},\mathbf{O}\\ \mathbf{B}_{1,2},\mathbf{B}_{1,1}\end{bmatrix}$, and

$$\begin{aligned}\mathbf{B}_{1,1}&=\text{blkdiag}\big[\mathbf{V}^T\left(\mathbf{I}_N-\mathbf{D}_1 d_{t,1}\right),\ldots,\\ &\qquad\mathbf{V}^T\left(\mathbf{I}_N-\mathbf{D}_M d_{t,M}\right)\big],\\ \mathbf{B}_{1,2}&=-\text{blkdiag}\big[\mathbf{V}^T\left(\dot{\mathbf{D}}_1 d_{t,1}+\mathbf{D}_1\dot{d}_{t,1}\right),\ldots,\\ &\qquad\mathbf{V}^T\left(\dot{\mathbf{D}}_M d_{t,M}+\mathbf{D}_M\dot{d}_{t,M}\right)\big]\end{aligned} \tag{18}$$

The WLS solution of (17), which minimizes the cost function $\boldsymbol{\varepsilon}_1^T\mathbf{W}_1\boldsymbol{\varepsilon}_1$ with respect to $\boldsymbol{\theta}$, is [20]

$$\hat{\boldsymbol{\theta}}=\left(\hat{\mathbf{G}}_1^T\mathbf{W}_1\hat{\mathbf{G}}_1\right)^{-1}\hat{\mathbf{G}}_1^T\mathbf{W}_1\hat{\mathbf{h}}_1 \tag{19}$$

where $\mathbf{W}_1$ is a symmetric positive definite matrix chosen here as

$$\mathbf{W}_1=\mathbb{E}\left[\boldsymbol{\varepsilon}_1\boldsymbol{\varepsilon}_1^T\right]^{-1}=\left(\mathbf{B}_1\mathbf{Q}_m\mathbf{B}_1^T\right)^{-1}, \tag{20}$$

Note that the weighting matrix $\mathbf{W}_1$ is dependent on the unknown nuisance parameters through $\mathbf{B}_1$. To implement the algorithm in the first stage, we first consider $\mathbf{W}_1=\mathbf{Q}_m^{-1}$ to obtain an initial estimate of the target position and velocity. The estimated values of $\hat{\mathbf{x}}_0$ and $\hat{\dot{\mathbf{x}}}_0$ are then applied to estimate the nuisance parameters and generate a more accurate value of $\mathbf{W}_1$. Repeating again the WLS solution (19) with the new weighting matrix results in a more accurate estimate of the target position and velocity.

Stage 2: In this stage, we refine the solution obtained in the first stage. Specifically, we provide estimates of errors in the estimation of the target position and the target velocity, $\Delta\mathbf{x}_0$ and $\Delta\dot{\mathbf{x}}_0$, and subtract them from the estimated target position and velocity (obtained in the first stage). Such a refinement can result in a substantial improvement in the performance of the proposed method.

We substitute the terms $\mathbf{x}_0=\hat{\mathbf{x}}_0-\Delta\mathbf{x}_0$, $\dot{\mathbf{x}}_0=\hat{\dot{\mathbf{x}}}_0-\Delta\dot{\mathbf{x}}_0$, $r_{i,j}=\hat{r}_{i,j}-\Delta r_{i,j}$ and $\dot{r}_{i,j}=\hat{\dot{r}}_{i,j}-\Delta\dot{r}_{i,j}$ into (7) and (8). Further, we approximate $d_{t,i}=\|\mathbf{x}_0-\mathbf{x}_{t,i}\|$ and $\dot{d}_{t,i}=\boldsymbol{\rho}_{\mathbf{x}_0,\mathbf{x}_{t,i}}^T(\dot{\mathbf{x}}_0-\dot{\mathbf{x}}_{t,i})$ in (7) and (8) with the first two terms of their corresponding Taylor series. That is

$$d_{t,i}=\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}-\Delta\mathbf{x}_0\|\approx\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|-\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\Delta\mathbf{x}_0, \tag{21}$$

$$\begin{aligned}\dot{d}_{t,i}&=\frac{(\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}-\Delta\mathbf{x}_0)^T\left(\hat{\dot{\mathbf{x}}}_0-\dot{\mathbf{x}}_{t,i}-\Delta\dot{\mathbf{x}}_0\right)}{\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}-\Delta\mathbf{x}_0\|}\\ &\approx\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\left(\hat{\dot{\mathbf{x}}}_0-\dot{\mathbf{x}}_{t,i}\right)-\dot{\boldsymbol{\rho}}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\Delta\mathbf{x}_0-\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\Delta\dot{\mathbf{x}}_0,\end{aligned} \tag{22}$$

where $\boldsymbol{\rho}_{\mathbf{a},\mathbf{b}}$ is given below (4) and $\dot{\boldsymbol{\rho}}_{\mathbf{a},\mathbf{b}}$ is the time derivative of $\boldsymbol{\rho}_{\mathbf{a},\mathbf{b}}$ defined as $\dot{\boldsymbol{\rho}}_{\mathbf{a},\mathbf{b}}=\left(\mathbf{I}-\frac{(\mathbf{a}-\mathbf{b})(\mathbf{a}-\mathbf{b})^T}{\|\mathbf{a}-\mathbf{b}\|^2}\right)(\dot{\mathbf{a}}-\dot{\mathbf{b}})/\|\mathbf{a}-\mathbf{b}\|$. As a result, (7) and (8), after neglecting the error terms higher than the linear ones, are expressed as follows

$$\begin{aligned}&\tfrac{1}{2}\left(\mathbf{x}_{t,i}^T\mathbf{x}_{t,i}-\mathbf{x}_{r,j}^T\mathbf{x}_{r,j}+\hat{r}_{i,j}^2\right)-(\mathbf{x}_{t,i}-\mathbf{x}_{r,j})^T\hat{\mathbf{x}}_0\\ &-\hat{r}_{i,j}\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|+(\mathbf{x}_{t,i}-\mathbf{x}_{r,j}+\hat{r}_{i,j}\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}})^T\Delta\mathbf{x}_0\\ &\approx\left(\hat{r}_{i,j}-\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|\right)\Delta r_{i,j},\end{aligned} \tag{23}$$

$$\begin{aligned}&\left(\dot{\mathbf{x}}_{t,i}^T\mathbf{x}_{t,i}-\dot{\mathbf{x}}_{r,j}^T\mathbf{x}_{r,j}+\hat{\dot{r}}_{i,j}\hat{r}_{i,j}\right)-(\dot{\mathbf{x}}_{t,i}-\dot{\mathbf{x}}_{r,j})^T\hat{\mathbf{x}}_0\\ &-(\mathbf{x}_{t,i}-\mathbf{x}_{r,j})^T\hat{\dot{\mathbf{x}}}_0-\hat{\dot{r}}_{i,j}\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|-\hat{r}_{i,j}\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\left(\hat{\dot{\mathbf{x}}}_0-\dot{\mathbf{x}}_{t,i}\right)\\ &+(\dot{\mathbf{x}}_{t,i}-\dot{\mathbf{x}}_{r,j}+\hat{\dot{r}}_{i,j}\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}+\hat{r}_{i,j}\dot{\boldsymbol{\rho}}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}})^T\Delta\mathbf{x}_0\\ &+(\mathbf{x}_{t,i}-\mathbf{x}_{r,j}+\hat{r}_{i,j}\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}})^T\Delta\dot{\mathbf{x}}_0\\ &\approx\left(\hat{\dot{r}}_{i,j}-\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\left(\hat{\dot{\mathbf{x}}}_0-\dot{\mathbf{x}}_{t,i}\right)\right)\Delta r_{i,j}\\ &\quad-\left(\hat{r}_{i,j}-\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|\right)\Delta\dot{r}_{i,j}.\end{aligned} \tag{24}$$

Applying (23) and (24) for all transmitters and receivers and combining them, we obtain

$$\boldsymbol{\varepsilon}_2=\hat{\mathbf{h}}_2-\hat{\mathbf{G}}_2\Delta\boldsymbol{\theta}, \tag{25}$$

where $\boldsymbol{\varepsilon}_2=\hat{\mathbf{B}}_2\Delta\mathbf{m}$, $\hat{\mathbf{h}}_2=\begin{bmatrix}\hat{\mathbf{h}}_{2,1}\\ \hat{\mathbf{h}}_{2,2}\end{bmatrix}$, $\hat{\mathbf{G}}_2=\begin{bmatrix}\hat{\mathbf{G}}_{2,1},\mathbf{O}\\ \hat{\mathbf{G}}_{2,2},\hat{\mathbf{G}}_{2,1}\end{bmatrix}$, $\hat{\mathbf{B}}_2=\begin{bmatrix}\hat{\mathbf{B}}_{2,1},\mathbf{O}\\ \hat{\mathbf{B}}_{2,2},\hat{\mathbf{B}}_{2,1}\end{bmatrix}$,

$$\begin{aligned}\hat{\mathbf{h}}_{2,1}(k)&=\tfrac{1}{2}\left(\mathbf{x}_{t,i}^T\mathbf{x}_{t,i}-\mathbf{x}_{r,j}^T\mathbf{x}_{r,j}+\hat{r}_{i,j}^2\right)\\ &\quad-(\mathbf{x}_{t,i}-\mathbf{x}_{r,j})^T\hat{\mathbf{x}}_0-\hat{r}_{i,j}\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|,\\ \hat{\mathbf{h}}_{2,2}(k)&=\left(\dot{\mathbf{x}}_{t,i}^T\mathbf{x}_{t,i}-\dot{\mathbf{x}}_{r,j}^T\mathbf{x}_{r,j}+\hat{\dot{r}}_{i,j}\hat{r}_{i,j}\right)\\ &\quad-(\dot{\mathbf{x}}_{t,i}-\dot{\mathbf{x}}_{r,j})^T\hat{\mathbf{x}}_0-(\mathbf{x}_{t,i}-\mathbf{x}_{r,j})^T\hat{\dot{\mathbf{x}}}_0\\ &\quad-\hat{\dot{r}}_{i,j}\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|-\hat{r}_{i,j}\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\left(\hat{\dot{\mathbf{x}}}_0-\dot{\mathbf{x}}_{t,i}\right),\\ \hat{\mathbf{G}}_{2,1}(k,:)&=-(\mathbf{x}_{t,i}-\mathbf{x}_{r,j}+\hat{r}_{i,j}\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}})^T,\\ \hat{\mathbf{G}}_{2,2}(k,:)&=-(\dot{\mathbf{x}}_{t,i}-\dot{\mathbf{x}}_{r,j}+\hat{\dot{r}}_{i,j}\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}\\ &\qquad+\hat{r}_{i,j}\dot{\boldsymbol{\rho}}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}})^T,\\ \hat{\mathbf{B}}_{2,1}(k,k)&=\hat{r}_{i,j}-\|\hat{\mathbf{x}}_0-\mathbf{x}_{t,i}\|,\\ \hat{\mathbf{B}}_{2,1}(k,k)&=\hat{\dot{r}}_{i,j}-\boldsymbol{\rho}_{\hat{\mathbf{x}}_0,\mathbf{x}_{t,i}}^T\left(\hat{\dot{\mathbf{x}}}_0-\dot{\mathbf{x}}_{t,i}\right),\end{aligned} \tag{26}$$

and $k=(i-1)N+j$. The WLS solution of (25) is [20]

$$\Delta\hat{\boldsymbol{\theta}}=\left(\hat{\mathbf{G}}_2^T\mathbf{W}_2\hat{\mathbf{G}}_2\right)^{-1}\hat{\mathbf{G}}_2^T\mathbf{W}_2\hat{\mathbf{h}}_2, \tag{27}$$

where $\mathbf{W}_2$ is the weighting matrix given by

$$\mathbf{W}_2=\left(\hat{\mathbf{B}}_2\mathbf{Q}_m\hat{\mathbf{B}}_2^T\right)^{-1}. \tag{28}$$

Finally, the refined solution of the target position and velocity is obtained via

$$\tilde{\boldsymbol{\theta}}=\hat{\boldsymbol{\theta}}-\Delta\hat{\boldsymbol{\theta}}, \tag{29}$$

where $\tilde{\boldsymbol{\theta}}$ denotes the final estimate of the target position and velocity $\boldsymbol{\theta}$.

IV. Performance Analysis

In this section, we aim to establish the efficient performance of the proposed method. To this end, we first compute the error covariance matrix of the proposed estimator. Then, we showcase that the performance of the proposed estimator can achieve the CRLB under mild Gaussian measurement noise conditions.

To derive the error covariance matrix of the proposed estimator, it is needed to compute $\tilde{\boldsymbol{\theta}} - E\left\{\tilde{\boldsymbol{\theta}}\right\}$. specifically, we express $\hat{\boldsymbol{\theta}}$ and $\Delta\hat{\boldsymbol{\theta}}$, the estimated values in the first and second stages, as $\boldsymbol{\theta} + \Delta\boldsymbol{\theta}$ and $\Delta\boldsymbol{\theta} + \delta\boldsymbol{\theta}$, respectively. Then, we have

$$\tilde{\boldsymbol{\theta}} - E\left\{\tilde{\boldsymbol{\theta}}\right\} = E\left\{\delta\boldsymbol{\theta}\right\} - \delta\boldsymbol{\theta}. \tag{30}$$

Subtracting $\Delta\boldsymbol{\theta}$ from both sides of (27), defining $\delta\boldsymbol{\theta} = \Delta\hat{\boldsymbol{\theta}} - \Delta\boldsymbol{\theta}$, and using (25), we obtain

$$\delta\boldsymbol{\theta} = \left(\hat{\mathbf{G}}_2^T\mathbf{W}_2\hat{\mathbf{G}}_2\right)^{-1}\hat{\mathbf{G}}_2^T\mathbf{W}_2\boldsymbol{\varepsilon}_2. \tag{31}$$

The direct computation of $E\left\{\delta\boldsymbol{\theta}\right\}$ is a rather intractable task as the error terms and noise exist in both $\hat{\mathbf{G}}_2$ and $\boldsymbol{\varepsilon}_2$. In the case of small measurement noise and error (i.e., when $\Delta r_{i,j} \ll r_{i,j}$ and $\Delta\dot{r}_{i,j} \ll \dot{r}_{i,j}$ for $i = 1,\ldots,M$ and $j = 1,\ldots,N$ and $\Delta\boldsymbol{\theta}(l) \ll \boldsymbol{\theta}(l)$ for $l = 1,\ldots,6$), the corresponding terms in $\hat{\mathbf{G}}_2$ and $\hat{\mathbf{B}}_2$ can be ignored. As a result, one can relate $\delta\boldsymbol{\theta}$ to the measurement noise vector $\Delta\mathbf{m}$ via a linear dependence as

$$\delta\boldsymbol{\theta} \approx \left(\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2\right)^{-1}\mathbf{G}_2^T\mathbf{W}_2\mathbf{B}_2\Delta\mathbf{m}. \tag{32}$$

It follows from (32) that $E\left\{\delta\boldsymbol{\theta}\right\} = 0$. Thus, the error covariance matrix of the proposed estimator can be approximately expressed as follows

$$\text{cov}(\tilde{\boldsymbol{\theta}}) = \text{cov}(\delta\boldsymbol{\theta}) \approx \left(\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2\right)^{-1}. \tag{33}$$

The CRLB of $\boldsymbol{\theta}$ equals the inverse of the Fisher information matrix. Under the Gaussian measurement noise model, it is simplified as [20]

$$\text{CRLB}(\boldsymbol{\theta}) = \left(\nabla_{\boldsymbol{\theta}}^T(\mathbf{m})\mathbf{Q}_m^{-1}\nabla_{\boldsymbol{\theta}}(\mathbf{m})\right)^{-1}. \tag{34}$$

where $\nabla_{\boldsymbol{\theta}}(\mathbf{m})$ denotes the partial derivative of the true measurement vector $\mathbf{m}$ with respect to the unknown vector $\boldsymbol{\theta}$ and can be expressed as follows

$$\nabla_{\boldsymbol{\theta}}(\mathbf{m}) = \begin{bmatrix}\mathbf{C} & \mathbf{O}\\ \dot{\mathbf{C}} & \mathbf{C}\end{bmatrix}, \tag{35}$$

where $\mathbf{C}$ and $\dot{\mathbf{C}}$ are matrices of size $MN \times 3$ in which the kth rows are given by $\mathbf{C}(k,:) = \boldsymbol{\rho}_{\mathbf{x}_0,\mathbf{x}_{t,i}}^T + \boldsymbol{\rho}_{\mathbf{x}_0,\mathbf{x}_{r,j}}^T$ and $\dot{\mathbf{C}}(k,:) = \dot{\boldsymbol{\rho}}_{\mathbf{x}_0,\mathbf{x}_{t,i}}^T + \dot{\boldsymbol{\rho}}_{\mathbf{x}_0,\mathbf{x}_{r,j}}^T$, where $\boldsymbol{\rho}_{\mathbf{a},\mathbf{b}}$ and $\dot{\boldsymbol{\rho}}_{\mathbf{a},\mathbf{b}}$ are given below (4) and (22), respectively, and $k = (i-1)N + j$ for $i = 1,\ldots,M$ and $j = 1,\ldots,N$. If we substitute $\mathbf{W_2}$ given by (28) in (33), we will obtain the following expression for the covariance matrix of the proposed estimator

$$\text{cov}(\tilde{\boldsymbol{\theta}}) \approx \left(\mathbf{G}_3^T\mathbf{Q}_m^{-1}\mathbf{G}_3\right)^{-1}, \tag{36}$$

where we have defined $\mathbf{G}_3 = \mathbf{B}_2^{-1}\mathbf{G}_2$. It can be noticed that the above expression for the covariance of the proposed estimator and the CRLB given by (34) have similar forms. In addition, if we form $\mathbf{G}_3$ using some straightforward mathematical manipulations, we obtain

$$\mathbf{G}_3 = \nabla_{\boldsymbol{\theta}}(\mathbf{m}). \tag{37}$$

As a result, we analytically confirm under the small Gaussian measurement noise that $\text{cov}(\tilde{\boldsymbol{\theta}}) \approx \text{CRLB}(\boldsymbol{\theta})$.

TABLE I: True Position and Velocity of Transmitters and Receivers (in SI units).

Tx no. i	$x_{t,i}$	$y_{t,i}$	$z_{t,i}$	$\dot{x}_{t,i}$	$\dot{y}_{t,i}$	$\dot{z}_{t,i}$
1	$R\cos(\pi/6)$	$R/2$	300	10	10	10
2	0	$2R$	250	20	0	0
3	$-R\cos(\pi/6)$	$R/2$	400	10	100	10
4	$-2R\cos(\pi/6)$	$-R$	100	20	15	10
5	0	$-R$	200	40	30	0
6	$2R\cos(\pi/6)$	$-R$	150	50	25	15
Rx no. j	$x_{r,j}$	$y_{r,j}$	$z_{r,j}$	$\dot{x}_{r,j}$	$\dot{y}_{r,j}$	$\dot{z}_{r,j}$
1	0	0	100	30	-20	20
2	$2R\cos(\pi/6)$	0	200	-30	10	20
3	$R\cos(\pi/6)$	$3R/2$	350	10	-20	10
4	$-R\cos(\pi/6)$	$3R/2$	250	10	20	30
5	$-2R\cos(\pi/6)$	0	400	-20	10	10
6	$-R\cos(\pi/6)$	$-3R/2$	150	20	-10	10
7	$R\cos(\pi/6)$	$-3R/2$	300	15	20	0

V. Simulations

The radar geometry used for numerical simulations is demonstrated in Fig. 1. We consider a MIMO radar with $M = 6$ transmitters and $N = 7$ receivers, whose positions and velocities are tabulated in Table I, where R is the side length of each hexagon depicted in Fig. 1 and is considered to be 1000 m. In simulations, we consider two targets located at $\mathbf{x}_0^{(1)} = [-0.5R, -0.5R, 0.5R]^T$m and $\mathbf{x}_0^{(2)} = [2R, 2R, 2R]^T$m, representing a near-field and a far-field target, respectively. The velocity of both targets are considered to be the same and equal to $\dot{\mathbf{x}}_0^{(1)} = \dot{\mathbf{x}}_0^{(2)} = [50, 50, 20]^T$m/s. The localization accuracy is assessed via the root mean squares error (RMSE) criterion, which is defined as $\text{RMSE}(\mathbf{x}_0) = \sqrt{\sum_{l=1}^{L}\left\|\hat{\mathbf{x}}_0^{(l)} - \mathbf{x}_0\right\|^2 / L}$ for position and $\text{RMSE}(\dot{\mathbf{x}}_0) = \sqrt{\sum_{l=1}^{L}\left\|\hat{\dot{\mathbf{x}}}_0^{(l)} - \dot{\mathbf{x}}_0\right\|^2 / L}$ for velocity, where $\hat{\mathbf{x}}_0^{(l)}$ and $\hat{\dot{\mathbf{x}}}_0^{(l)}$ are the estimates of $\mathbf{x}_0$ and $\dot{\mathbf{x}}_0$ at the lth Monte-Carlo run, respectively, and $L = 1000$ is the number of runs. The noise covariance matrix is considered as $\mathbf{Q}_m = \text{blkdiag}(\sigma_r\mathbf{J}_1, \sigma_{\dot{r}}\mathbf{J}_1)$, where $\sigma_{\dot{r}} = 0.01\sigma_r$, $\mathbf{J}_1$ is an $MN \times MN$ diagonal matrix with diagonal elements equal to one. In the simulation, σ_r^2 varies from 10^{-4} to 10^2. The position and velocity RMSE of the different estimators versus σ_α^2 for both targets, $\mathbf{x}_0^{(1)}$ and $\mathbf{x}_0^{(2)}$, are represented in Fig. 2 (a) and (b) and Fig. 3 (a) and (b), respectively. As shown in these figures, the proposed estimator outperforms the other algorithms in both cases. It can be seen that the proposed method can attain the CRLB performance for both position and velocity up to relatively high noise levels.

VI. Conclusion

We proposed an efficient algebraic TSWLS method for moving target localization in distributed MIMO radar systems.

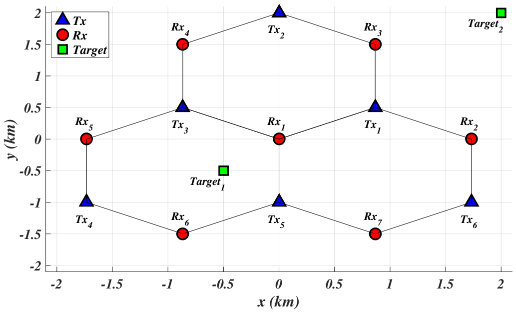

Fig. 1: Geometry of the distributed MIMO radar system in the x-y plane.

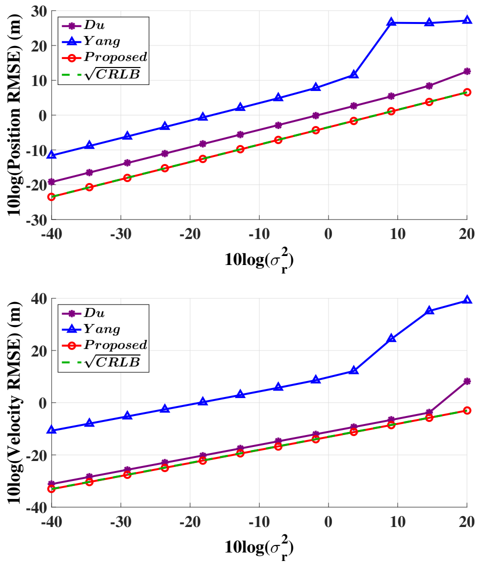

Fig. 2: Performance comparison of different localization estimators for the first target $\mathbf{x}_0^{(1)}$.

Simulation results demonstrated that the proposed approach achieves near-Cramér-Rao Lower Bound (CRLB) accuracy under mild noise conditions and outperforms existing localization methods in both position and velocity estimation.

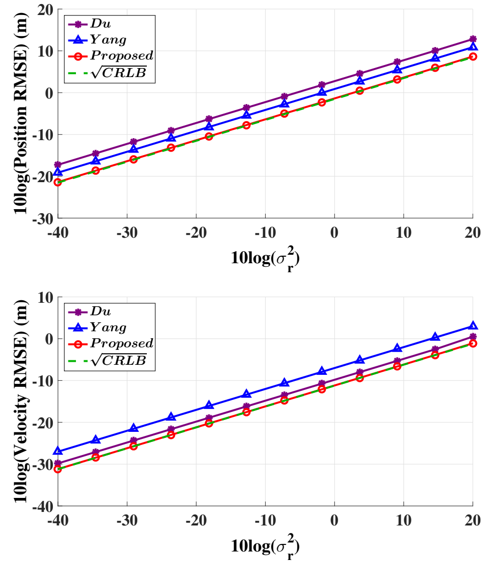

Fig. 3: Performance comparison of different localization estimators for the second target $\mathbf{x}_0^{(2)}$.

References

[1] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 106–114, sep 2007.

[2] A. Haimovich, R. Blum, and L. Cimini, "MIMO radar with widely separated antennas," *IEEE Signal Process. Mag.*, vol. 25, no. 1, pp. 116–129, 2008.

[3] A. Ajorloo, A. Amini, and M. H. Bastani, "A compressive sensing based colocated MIMO radar power allocation and waveform design," *IEEE Sensors Journal*, pp. 1–1, 2018.

[4] M. Radmard, M. M. Chitgarha, M. N. Majd, and M. M. Nayebi, "Antenna placement and power allocation optimization in MIMO detection," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 2, pp. 1468–1478, 2014.

[5] A. Noroozi, R. Amiri, M. M. Nayebi, and A. Farina, "Efficient closed-form solution for moving target localization in MIMO radars with minimum number of antennas," *IEEE Transactions on Signal Processing*, 2020.

[6] A. Ajorloo, A. Amini, E. Tohidi, M. H. Bastani, and G. Leus, "Antenna placement in a compressive sensing-based colocated MIMO radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 56, no. 6, pp. 4606–4614, 2020.

[7] H. Li, Z. Wang, J. Liu, and B. Himed, "Moving target detection in distributed mimo radar on moving platforms," *IEEE Journal of Selected Topics in Signal Processing*, vol. 9, no. 8, pp. 1524–1535, 2015.

[8] A. Noroozi and M. A. Sebt, "Algebraic solution for three-dimensional tdoa/aoa localisation in multiple-input–multiple-output passive radar," *IET Radar, Sonar & Navigation*, 2017.

[9] M. R. Jabbari, M. R. Taban, and S. Gazor, "Moving target localization in distributed sensor networks via nuisance variables elimination," in *2024 IEEE Canadian Conference on Electrical and Computer Engineering (CCECE)*. IEEE, 2024, pp. 252–256.

[10] G. Zhang, W. Yi, P. K. Varshney, and L. Kong, "Direct target localization with quantized measurements in noncoherent distributed MIMO radar systems," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 61, pp. 1–18, 2023.

[11] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R. A. Valenzuela, "Spatial diversity in radars—models and detection performance," *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 823–838, 2006.

[12] R. Amiri, F. Behnia, and H. Zamani, "Efficient 3-d positioning using time-delay and aoa measurements in mimo radar systems," *IEEE Communications Letters*, vol. 21, no. 12, pp. 2614–2617, 2017.

[13] A. Noroozi and M. A. Sebt, "Target localization from bistatic range measurements in multi-transmitter multi-receiver passive radar," *IEEE Signal. Process. Lett.*, vol. 22, no. 12, pp. 2445–2449, dec 2015.

[14] R. Amiri and F. Behnia, "An efficient weighted least squares estimator for elliptic localization in distributed mimo radars," *IEEE Signal Processing Letters*, vol. 24, no. 6, pp. 902–906, 2017.

[15] A. Noroozi and M. A. Sebt, "Target localization in multistatic passive radar using svd approach," *IEEE Transactions on Aerospace and Electronic Systems*, vol. PP, no. 99, pp. 1–1, 2017.

[16] A. Noroozi, A. H. Oveis, and M. A. Sebt, "Iterative target localization in distributed mimo radar from bistatic range measurements," *IEEE Signal Processing Letters*, vol. 24, no. 11, pp. 1709–1713, 2017.

[17] A. Noroozi, M. A. Sebt, S. M. Hosseini, R. Amiri, and M. M. Nayebi, "Closed-form solution for elliptic localization in distributed MIMO radar systems with minimum number of sensors," *IEEE Transactions on Aerospace and Electronic Systems*, 2020.

[18] Y. Du and P. Wei, "An explicit solution for target localization in noncoherent distributed MIMO radar systems," *IEEE Signal Processing Letters*, vol. 21, no. 9, pp. 1093–1097, 2014.

[19] H. Yang and J. Chun, "An improved algebraic solution for moving target localization in noncoherent MIMO radar systems," *IEEE Trans. Signal Process.*, vol. 64, no. 1, pp. 258–270, Jan 2016.

[20] S. M. Kay, *Fundamentals of Statistical Processing, Volume I: Estimation Theory*. Prentice Hall, 1993.