# Leak Detection in Water Distribution Networks Using Topological Signal Processing

Tiziana Cattai[1], Stefania Sardellitti[2], Stefania Colonnese[1], Francesca Cuomo[1], Sergio Barbarossa[1]
[1] Department of Information Engineering, Electronics, and Telecommunications, Sapienza University of Rome
[2] Dept. of Engineering and Sciences, Universitas Mercatorum, Rome, Italy
e-mails: tiziana.cattai@uniroma1.it, stefania.sardellitti@unimercatorum.it, stefania.colonnese@uniroma1.it, francesca.cuomo@uniroma1.it, sergio.barbarossa@uniroma1.it

***Abstract*—The efficient management of water resources is crucial in today's society, as growing populations, climate change, and increasing water scarcity demand improved monitoring and assessment of Water Distribution Networks (WDNs). In this paper, we propose a dynamic model for the analysis of water flow in WDNs that accounts for key real-world factors, such as water demands and leakages, with the ultimate goal of leakage detection. Traditional monitoring approaches struggle with the complexity of WDNs, particularly when relying on limited sensor data. To address this challenge, we leverage Topological Signal Processing (TSP) to model and analyze water flow as high-order signals defined on the edges of higher-order topological structures such as cell complexes. By incorporating these higher-order topological structures, we develop a learning-based approach to reconstruct the dynamics of the water flows from observations of a reduced number of sensors. Then, we propose an anomaly detection algorithm designed to precisely identify leakages by formulating an optimization problem that minimizes the recovery error to detect sparse leakages. Performance results demonstrate the effectiveness of topological-based learning for efficiently monitoring water distribution network leakages.**

***Index Terms*—Topological signal processing, water distribution networks, anomaly detection.**

## I. Introduction

The growing global challenge of water scarcity, caused by climate change and increasing population demands, underscores the urgent need for efficient water monitoring and leak detection in Water Distribution Networks (WDNs) [1], [2]. Identifying leaks is particularly challenging in realistic scenarios, where sensor coverage is limited and direct flow measurements across the entire network are not available [3], [4]. The complexity of WDNs, combined with the variability in data collection methods, further complicates the task, making traditional monitoring approaches—such as manual inspections or simplified models—inefficient, labor-intensive, and prone to inaccuracies [5]. Developing methods that enable accurate leak detection with sparse sensor data is therefore essential to ensure a sustainable and reliable water supply.

To address this challenge, data-driven methods have been explored for leak detection in WDNs. These approaches, often based on machine learning and graph neural networks, leverage historical data to identify patterns associated with anomalies [6], [7]. However, their applicability and effectiveness are limited by the high computational cost and the need for large labeled datasets for training, which are not always available in the context of WDNs. In contrast, model-based approaches have shown promising results in identifying network anomalies by leveraging physical principles and mathematical models to estimate water flow and pressure [8]. These methods, however, often lack the capability to precisely localize leaks within the network, which is a critical limitation for effective usability in WDNs.

In this paper, we introduce a novel dynamic model for WDNs based on Topological Signal Processing (TSP) a tool to describe and process signals defined over high-order networks [9], [10]. While traditional Graph Signal Processing (GSP) techniques represent signals on graph nodes and capture pairwise interactions between nodes [11], TSP extends this framework by enabling the analysis of signals on more intricate topological structures, such as cell complexes [9], [12]. This higher-order data representation allows for a richer characterization of network dynamics, making it particularly suitable for applications where interactions extend beyond direct node-to-node connections [13].

Here, we propose a dynamic model for real-time monitoring of WDNs for leak detection. This model extends the capability of static approaches by incorporating temporal dynamics into the analysis, allowing for the characterization of transient water demands and persistent leakages. In addition, cost limitations and the inaccessibility of certain locations often restrict the number and placement of sensors, preventing the widespread deployment of IoT devices. Consequently, it is essential to reconstruct the flow signals in unmeasured locations [4], [14] and, for this aim, we adopt graph sampling theory [15]. Therefore, we reconstruct the edge signals from a subset of observed samples and we formulate an optimization problem to detect water leakages. We test our method on simulated WDNs with water demands and random leakages, and we demonstrate that it is able to reconstruct the flow across the entire network and accurately detect leakages.

## II. Topological Signal Processing for WDNs

In this section, we provide an overview of TSP tools [9], [10], which serve as bases for developing a dynamic topology-based model for water flows in WDNs.

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A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ can be simply associated with a water distribution network, with $\mathcal{V} = \{v_1, \ldots, v_N\}$ the set of $N$ nodes and $\mathcal{E} = \{e_{ij}\}_{i,j\in\mathcal{V}}$ the set of $E$ edges. The generic $e_{ij}$ is equal to 1, if there is a link (a pipe) connecting node $i$ and node $j$, otherwise it is 0. An important matrix used to extract fundamental properties of the graph is the Laplacian matrix, which can be written as $\mathbf{L}_0 = \mathbf{B}_1\mathbf{B}_1^T$ with $\mathbf{B}_1$ the $N \times E$ incidence matrix of the graph $\mathcal{G}$. More specifically, given an orientation of the edges of the graph, the entries of $\mathbf{B}_1$ are defined as $B_1(i,j) = 0$ if $e_{ij} = 0$, $B_1(i,j) = 1$ if node $i$ is the tail of the edge $e_{ij}$ and $B_1(i,j) = -1$ if node $i$ is instead the head of $e_{ij}$.

Graphs are simple topological spaces that are able to capture pairwise relations between data associated with their vertices through the presence of edges. To incorporate higher-order relationships that may exist among data associated with the edges of a graph, like flow data, higher-order topological spaces, such as simplicial or cell complexes, need to be considered. Topological Signal Processing (TSP) has recently been introduced to analyze and process signals defined over simplicial and cell complexes [9], [10], [12].

A cell complex $\mathcal{C}$ is composed of a set of abstract elements, known as cells, characterized by a bounding relationship and a dimension function, which fulfill the transitivity and monotonicity properties [9]. A cell of dimension $n$ is referred to as an $n$-cell: vertices correspond to 0-cells, edges to 1-cells, and polygons of any order are classified as 2-cells.

The boundary of an $n$-dimensional cell consists of all lower-dimensional cells that define its enclosure. A cell complex $\mathcal{C}$ is considered $K$-dimensional if the highest dimension among its cells does not exceed $K$. Under these definitions, simplicial complexes can be seen as a specialized class of cell complexes, where each $k$-cell is formed by exactly $k+1$ vertices. The structure of $\mathcal{C}$ is typically described through its incidence matrices, which capture the relationships between $k$-cells and the corresponding $(k-1)$-cells. The concept of orientation in $\mathcal{C}$ incorporates both the orientation of individual cells and their incidence relationships [9]. Let $c_i^k$ represent the $i$-th cell of order $k$. If $c_i^{k-1} \prec_b c_j^k$, we say that $c_i^{k-1}$ is lower incident to $c_j^k$. Two $k$-order cells are considered lower adjacent if they share a common $(k-1)$-dimensional face, whereas they are upper adjacent if they both serve as faces of a $(k+1)$-dimensional cell. Defining an orientation of $\mathcal{C}$, its structural relationships up to order $K$ are fully described by the set of incidence matrices $\mathbf{B}_k$, for $k = 1, \ldots, K$. These matrices, also called boundary matrices, encode the connectivity between $k$-cells and their corresponding $(k-1)$-cells. They are formally defined as:

$$B_k(i,j) = \begin{cases} 0, & \text{if } c_i^{k-1} \not\prec_b c_j^k \\ 1, & \text{if } c_i^{k-1} \prec_b c_j^k \text{ and } c_i^{k-1} \sim c_j^k \\ -1, & \text{if } c_i^{k-1} \prec_b c_j^k \text{ and } c_i^{k-1} \nsim c_j^k \end{cases}. \quad (1)$$

We use the notation $c_i^{k-1} \sim c_j^k$ in the case of aligned orientations of $c_i^{k-1}$ and $c_j^k$, whereas $c_i^{k-1} \nsim c_j^k$ in case of opposite orientations. To characterize the $K$-dimensional cell complex, we adopt the combinatorial Laplacian matrices [16], defined as:

$$\begin{aligned} &\mathbf{L}_0 = \mathbf{B}_1\mathbf{B}_1^T, \\ &\mathbf{L}_k = \mathbf{B}_k^T\mathbf{B}_k + \mathbf{B}_{k+1}\mathbf{B}_{k+1}^T \quad \text{for } k = 1, \ldots, K-1 \\ &\mathbf{L}_K = \mathbf{B}_K^T\mathbf{B}_K \end{aligned} \quad (2)$$

with $\mathbf{L}_{k,d} = \mathbf{B}_k^T\mathbf{B}_k$ and $\mathbf{L}_{k,u} = \mathbf{B}_{k+1}\mathbf{B}_{k+1}^T$ representing, respectively, the lower and upper Laplacians.

An edge signal $\mathbf{x}$ over a cell complex $\mathcal{C}$ is defined as a function that assigns real numbers to each edge, i.e., $\mathbf{x} : \mathcal{E} \to \mathbb{R}$, with $|\mathcal{E}| = E$. The Cell Fourier Transform of an edge signal $\mathbf{x} \in \mathbb{R}^E$ has been defined as the projection of $\mathbf{x}$ onto the space spanned by the eigenvectors of $\mathbf{L}_1$, i.e. $\bar{\mathbf{x}} := \mathbf{U}_1^T\mathbf{x}$ [9]. An edge signal $\mathbf{x}$ is said to be bandlimited, with bandwidth $K$, if it admits a sparse representation, i.e. it can be represented using only $K$ eigenvectors of $\mathbf{L}_1$, i.e. as $\mathbf{x} = \mathbf{U}_{1,\mathcal{K}}\bar{\mathbf{x}}$, where $\mathbf{U}_{1,\mathcal{K}}$ is a matrix whose $K$ columns are the eigenvectors with indexes in the frequency set $\mathcal{K}$, with $|\mathcal{K}| = K$. If the observations of a bandlimited signal of order $K$ are available only over a subset $\mathcal{S}$ of edges, the edge signal can be recovered from a subset of observed samples $\mathbf{x}_S$ using the interpolation formula [10], [15]:

$$\hat{\mathbf{x}} = \left(\mathbf{I} - \bar{\mathbf{D}}_{\mathcal{S}}\mathbf{U}_{1,\mathcal{K}}\mathbf{U}_{1,\mathcal{K}}^T\right)^{-1}\mathbf{x}_S, \quad (3)$$

where $\mathbf{x}_S$ is a signal equal to $\mathbf{x}$ on the subset $\mathcal{S}$ and zero outside, $\mathbf{D}_{\mathcal{S}}$ is the diagonal selection matrix with diagonal entries equal to 1 for the edges in the sample set $\mathcal{S}$ and $\bar{\mathbf{D}}_{\mathcal{S}} = \mathbf{I} - \mathbf{D}_{\mathcal{S}}$. We provided that the number of samples is at least equal to the bandwidth $K$ [15]. This formulation ensures an optimal reconstruction of the missing edge values by leveraging the spectral properties of the first-order Laplacian.

## III. Dynamic water monitoring for Leak Detection

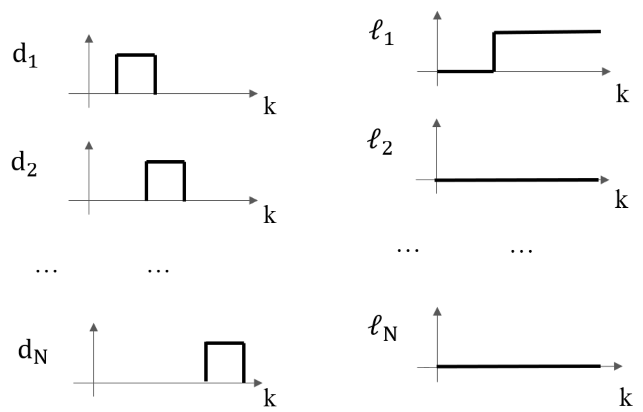
With the objective of identifying and characterizing water leakage in a water distribution network, it is essential to introduce a dynamic model of signals in a WDN.

Let us denote with $\mathbf{x}[k]$ the vector whose entries represent the flow on the edges of the cell complex at time $k$. The cell complex is built by filling all the polygonal cells in the water distribution network. Then, we model the dynamic time evolution of the flows as:

$$\mathbf{x}[k+1] = \mathbf{M}\mathbf{x}[k] + \mathbf{u}[k] \quad (4)$$

where the matrix $\mathbf{M}$ governs the system dynamics on the topology structure. In our model, we assume $\mathbf{M}$ equal to the mask of the first-order Laplacian $\mathbf{L}_1$, as it describes lower and upper adjacencies among edges. Therefore, the dynamic evolution of the water flows over time inherently incorporates the topological structure of the network.

To avoid stability issues, the matrix $\mathbf{M}$ is normalized by its maximum eigenvalue. The normalized matrix is then used in the model to simulate the flow of water through the network, incorporating the effects of leaks and the variation of water demands. By integrating these elements, we model the evolving interactions within the water distribution network, enabling the simulation of the flow dynamics under different conditions.




(a) (b)

Fig. 1. (a) Water demands associated with edges: sparse in time and randomly distributed; (b) water leakages associated with edges: sparse, as they occur on a small percentage of edges, but persistent over time.

The water flow $\mathbf{x}[k+1]$ at time instant $k+1$ incorporates the effects of water demands $\mathbf{d}[k]$ and leakages $\boldsymbol{\ell}[k]$ through the additive term $\mathbf{u}[k] = \mathbf{d}[k] + \boldsymbol{\ell}[k]$. Differently from other approaches where the demands are associated with node-based water consumption, here we express them as edge flows, ensuring a consistent edge-based representation for the entire system. These two terms have different characteristics. Demands are sporadic in time, since they last for a short duration on the time scale and appear on random edges. This corresponds, for example, to opening a faucet for a few minutes or using an appliance at another point in the network. Then, demands are characterized by their limited duration and random occurrence in the network. On the other hand, leaks may start at random moments but, once they occur, they are persistent (until an intervention is made to stop the leakage) and, statistically, they occur on a few edges of the complex. They are therefore sparse across the edges but persistent over time. This is precisely the property that we want to exploit to detect the leaks. This is illustrated in Fig. 1, where the behavior of the two terms of the model $\mathbf{d}[k+1]$ and $\boldsymbol{\ell}[k+1]$ is shown in plots (a) and (b), respectively.

### A. *Identifying anomalies on WDNs*

Leak detection in WDNs is crucial for minimizing water loss, reducing operational costs, and ensuring a reliable water supply [17]. Early detection helps prevent infrastructure damage, preserves resources, and supports environmental sustainability by minimizing waste. In order to identify water leakage, we formulate an optimization problem using the dynamic model in (4), able to simultaneously identify water demands and leakages through the term $\mathbf{u}[k]$. Therefore, we solve the following optimization problem:

$$\begin{aligned} \min_{\{\mathbf{u}[k]\}_{k=1}^{L}} \quad & \sum_{k=1}^{L} \parallel \mathbf{x}[k+1] - \mathbf{M}\mathbf{x}[k] - \mathbf{u}[k] \parallel_2^2 + \lambda \parallel \mathbf{u}[k] \parallel_1 \\ & \mathbf{u}[k] \leq \mathbf{0}, \ \forall k, \end{aligned} \tag{5}$$

Fig. 2. Cell complex representation of L-town.

where $L$ represents the length of the time window. This convex problem aims to simultaneously identifying water demands and leakages in the network. The first term in (5) quantifies the deviation between the observed flow signal at time $k$ and the expected flow signal based on the previous time step $k-1$. This term models the difference between the actual measurements and the predicted values considering the diffusion term $\mathbf{M}$. Our goal is to minimize this error, ensuring that the reconstructed flow $\mathbf{x}[k]$ closely follows the network's model behavior. Note that the obtained discrepancy corresponds to the anomalies in the diffusion process, that are the leakages and water demands. The second $l_1$-norm term in the objective function forces sparsity on the entries of the vector $\mathbf{u}[k]$. The non-negative penalty coefficient $\lambda$ controls the balance between the data-fitting term and the sparsity term.

We now present the results obtained using the proposed dynamic water monitoring model for leak detection on synthetic data from the L-town [18]. We start assuming the ideal (non realistic) case where the flows are observable over each link. Later on, we will remove this hypothesis and we will consider the more realistic scenario with sparse sensor measurements.

We report the cell complex $\mathcal{C}$ of the associated network in Fig. 2 [19]. Here, we introduced four leaks at specific edge locations in the network, with indices 200, 245, 500, and 650. These leaks were initiated at different time instants: 50, 20, 30, and 70, respectively, each with a constant intensity. Additionally, water demands were randomly applied to $5\%$ of the network edges. Each demand lasted for a fixed duration and intensity. The edges affected by these demands were randomly selected from the network. The results are visualized in Fig. 3, which shows the signal $\mathbf{u}[k]$ reconstructed using (5). From a visual inspection of this figure, it is possible to see that persistent values are derived in correspondence of the edges with leakages, which are represented in dark red stars. In order to identify leakages, we define the edge signal $\mathbf{s}_u = \sum_k |\mathbf{u}[k]|$ and represent its entries with blue lines in Fig. 4. We indicate the edges with leakage using red stars, while the horizontal black line denotes the threshold used to identify the leakages. Specifically, the values of $\mathbf{s}_u$ exceeding the threshold identify edges affected by leaks.

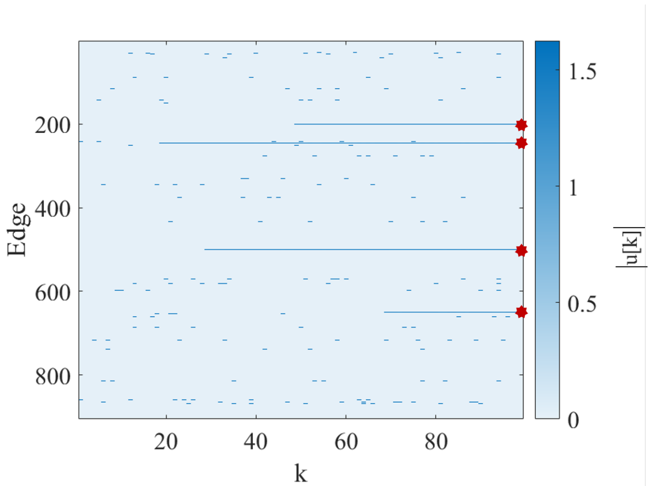



Fig. 3. Reconstructed $|\mathbf{u}[k]|$. Colors are associated with $\mathbf{u}[k]$ value at each edge and each time point $k$. In red stars we have leak positions.

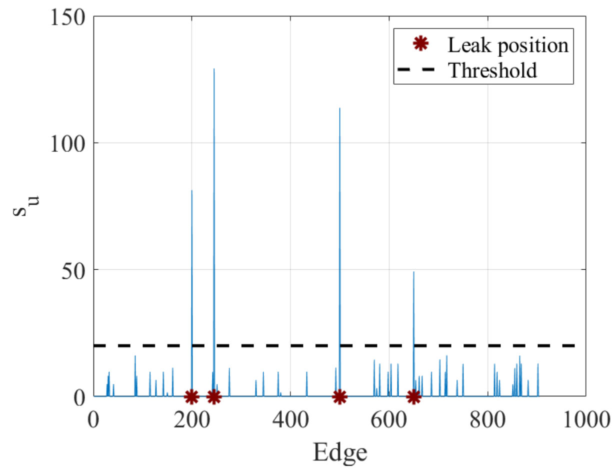



Fig. 4. Results of $\mathbf{s}_u$ in blue line. We represent the edges with leakage using red stars, while the horizontal black line represents an arbitrary threshold that can be used to identify the leakages.

*B. Identifying anomalies on WDNs from a subset of measurements*

In this section, we analyze the more realistic scenario where only sparse sensor measurements from a subset of graph edges are available. Our goal is to detect leakages under these conditions. Specifically, we aim to reconstruct the signal from a limited set of observations while simultaneously identifying potential leakages in the system.

Given the cell complex $\mathcal{C}$ associated with the WDN, we assume that the flow vector $\mathbf{x}[k]$ can be represented as a bandlimited signal of order $K$, meaning that $\mathbf{x}[k]$ can be expressed as the linear combination of $K$ eigenvectors. The optimal sensor locations, for a given set of measurements, are selected using the max-det method proposed in [11], assuming $|\mathcal{S}| \geq K$. The edge values on unmeasured locations are then reconstructed using (3) by obtaining the estimated edge flows $\hat{\mathbf{x}}[k]$.

**Algorithm 1** Leak Detection with sparse sensor measurements

**Input:** Cell complex structure $\mathcal{C}$, sampled edge measurements $\mathbf{x}_S[k]$, number of time steps $L$, threshold $\tau$
**Output:** Indices of detected leakages $\mathcal{L}$
**Step 1: Edge Value Sampling and Reconstruction**
Define optimal sensor locations $\mathcal{S}$ ensuring $|\mathcal{S}| \geq K$
**for** $k = 1$ to $L$ **do**
    Reconstruct edge flows $\hat{\mathbf{x}}[k]$ using interpolation:

$$\hat{\mathbf{x}}[k] = \left(\mathbf{I} - \bar{\mathbf{D}}_{\mathcal{S}} \mathbf{U}_{1,\mathcal{K}} \mathbf{U}_{1,\mathcal{K}}^T\right)^{-1} \mathbf{x}_S[k].$$

**end for**
**Step 2: Anomaly Detection**
Solve the optimization problem to obtain $\mathbf{u}[k]$:

$$\min_{\{\mathbf{u}[k]\}_{k=1}^{L}} \sum_{k=1}^{L} \| \mathbf{x}[k+1] - \mathbf{M}\mathbf{x}[k] - \mathbf{u}[k] \|_2^2 + \lambda \| \mathbf{u}[k] \|_1$$

$$\mathbf{u}[k] \leq \mathbf{0}, \ \forall k$$

**Step 3: Leakage Localization**
Compute cumulative leakage score: $\mathbf{s}_u = \sum_k |\mathbf{u}[k]|$
Set a detection threshold $\tau$
Identify leakage indices:

$$\mathcal{L} = \{ i \in (1, \ldots, \mathcal{E}) \mid \mathbf{s}_u(i) > \tau \}$$

**Return:** Indices of detected leakages $\mathcal{L}$

These flows are replaced for each time instant into the developed optimization problem (5) that accounts for the network's diffusion-like behavior and anomalies. Given the solution $\mathbf{u}[k]$ of (5), persistent deviations over time indicate the presence of a leak, while sporadic deviations may correspond to normal water demands.

To objectively evaluate the presence of leakages in the system, we firstly analyze the total leakage contribution at each edge over time by defining $\mathbf{s}_u = \sum_k |\mathbf{u}[k]|$. Then, with the goal to determine whether an edge should be classified as a leakage point, we define a detection threshold. Different strategies can be used to select the threshold. For instance, a statistical approach can be employed, modeling the network under leakage-free conditions with random scenarios that include only demands. In this way, the false alarm probability can be fixed to control the trade-off between true positive and negative in leakage detection. Any edge with a leakage value exceeding this threshold can be considered a detected leakage. To quantify the effectiveness of the detection, we compute the F1-score for each run, which balances precision and recall, and then we average it across the $N_{run}$ independent repetitions of the simulation. The details of the proposed algorithm are summarized in Algorithm 1.

To evaluate the proposed methodology, we implement a simulation focusing on leak detection with a limited subset of flow measurements. The simulation considers a time horizon of 100 time steps. Leaks are introduced on a 1% of the nodes, in random positions and with random starting times. The leak intensity is fixed throughout the simulation.

Additionally, demands were introduced at 5% of the nodes, with their locations chosen randomly. These demands have a fixed duration and intensity. To ensure statistical robustness and obtain results independent of the specific leakage positions relative to the sampled points, we perform $N_{run}$ repetitions of the simulation, averaging the outcomes across these runs. For the simulation here, we set $L = 10$. These analyses are repeated for different values of the number of available samples, ranging from 100 to the total number of 900. The goal is to analyze how the performance of the leak detection method varies as a function of the number of available samples, assessing its sensitivity to the amount of data collected. It is important to notice that as the number of samples increases the method is able to account for a larger bandwidth, under the recovery condition $|\mathcal{S}| > |\mathcal{K}|$. This implies that the signal can be more accurately recovered since we have more samples, and then we can use more eigenvectors to represent the signal.

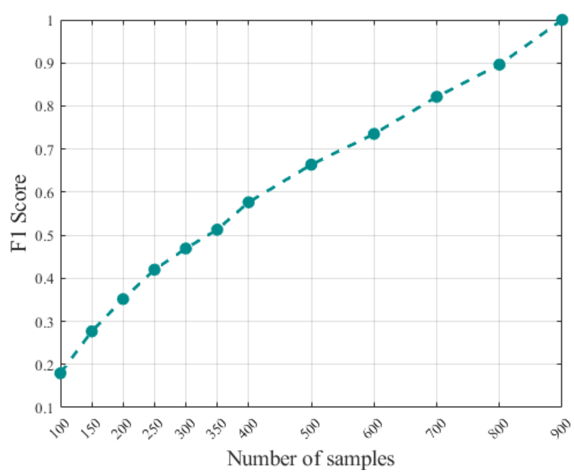



Fig. 5. Average F1-score as a function of the number of available samples.

To test the leakage detection performance, in Fig. 5, we report the average F1-score across $N_{run}$ independent simulations, as a function of the number of available samples by fixing the signal bandwidth. The plot clearly shows an increasing trend: as the number of samples grows, the performance of the leak detection method improves as well. This indicates that having more measurements leads to a more accurate identification of leakages. Eventually, when the total number of samples is available, the detection reaches its optimal performance, achieving perfect identification of the leakage points.

## IV. Conclusions

In this paper, we introduced a novel dynamic topology-based learning approach for leak detection in Water Distribution Networks. By leveraging Topological Signal Processing, we modeled water flow dynamics using high-order representations that extend the traditional graph-based frameworks. This approach allowed us to efficiently reconstruct flow signals from a limited number of sensor measurements of edge flows and accurately identifying leakages within the network. Our method combines a dynamic modeling framework with an optimization-based anomaly detection strategy, distinguishing between normal water demands and persistent leakages. The experimental results demonstrated that our approach successfully detects leaks even in scenarios with sparse sensor data. Moreover, the F1-score analysis highlighted that the performance of the leak detection method improves as the number of available measurements increases, reaching optimal detection when the full dataset is available. These findings confirm the potential of our dynamic topology-based learning for real-time WDN monitoring, offering an efficient solution for anomaly detection.

## References


[1] J. Liu, H. Yang, S. N. Gosling, M. Kummu, M. Flörke, S. Pfister, N. Hanasaki, Y. Wada, X. Zhang, C. Zheng, *et al.*, "Water scarcity assessments in the past, present, and future," *Earth's future*, vol. 5, no. 6, pp. 545–559, 2017.

[2] A. Pagano, D. Garlisi, F. Giuliano, T. Cattai, and F. C. Sapienza, "Swi-feed: Smart water iot framework for evaluation of energy and data in massive scenarios," in *2024 IFIP Networking Conference (IFIP Networking)*, pp. 583–585, IEEE, 2024.

[3] V. A. Tzanakakis, N. V. Paranychianakis, and A. N. Angelakis, "Water supply and water scarcity," *Water*, vol. 12, no. 9, p. 2347, 2020.

[4] T. Cattai, S. Colonnese, D. Garlisi, A. Pagano, and F. Cuomo, "Graphsmart: a method for green and accurate iot water monitoring," *ACM Transactions on Sensor Networks*, vol. 20, no. 6, pp. 1–32, 2024.

[5] Z. Liu and Y. Kleiner, "State of the art review of inspection technologies for condition assessment of water pipes," *Measurement*, vol. 46, no. 1, pp. 1–15, 2013.

[6] G. Ö. Garðarsson, F. Boem, and L. Toni, "Graph-based learning for leak detection and localisation in water distribution networks," *IFAC-PapersOnLine*, vol. 55, no. 6, pp. 661–666, 2022.

[7] W. Wu, X. Pan, Y. Kang, Y. Xu, and L. Han, "To feel the spatial: Graph neural network-based method for leakage risk assessment in water distribution networks," *Water*, vol. 16, no. 14, p. 2017, 2024.

[8] C. Liu and E. Isufi, "Hodge-aware matched subspace detectors," in *2024 32nd European Signal Processing Conference (EUSIPCO)*, pp. 817–821, IEEE, 2024.

[9] S. Sardellitti and S. Barbarossa, "Topological signal processing over generalized cell complexes," *IEEE Transactions on Signal Processing*, vol. 72, pp. 687–700, 2024.

[10] S. Barbarossa and S. Sardellitti, "Topological signal processing over simplicial complexes," *IEEE Transactions on Signal Processing*, vol. 68, pp. 2992–3007, 2020.

[11] A. Sandryhaila and J. M. Moura, "Discrete signal processing on graphs," *IEEE Transactions on Signal Processing*, vol. 61, no. 7, pp. 1644–1656, 2013.

[12] S. Barbarossa and S. Sardellitti, "Topological signal processing: Making sense of data building on multiway relations," *IEEE Signal Processing Magazine*, vol. 37, no. 6, pp. 174–183, 2020.

[13] T. Cattai, S. Sardellitti, S. Colonnese, F. Cuomo, and S. Barbarossa, "Physics-informed topological signal processing for water distribution network monitoring," *arXiv preprint arXiv:2505.07560*, 2025.

[14] Z. Wei, A. Pagani, G. Fu, I. Guymer, W. Chen, J. McCann, and W. Guo, "Optimal sampling of water distribution network dynamics using graph Fourier transform," *IEEE Transactions on Network Science and Engineering*, vol. 7, no. 3, pp. 1570–1582, 2019.

[15] M. Tsitsvero, S. Barbarossa, and P. Di Lorenzo, "Signals on graphs: Uncertainty principle and sampling," *IEEE Transactions on Signal Processing*, vol. 64, no. 18, pp. 4845–4860, 2016.

[16] D. Horak and J. Jost, "Spectra of combinatorial Laplace operators on simplicial complexes," *Adv. Math.*, vol. 244, pp. 303–336, 2013.

[17] S. El-Zahab and T. Zayed, "Leak detection in water distribution networks: an introductory overview," *Smart Water*, vol. 4, no. 1, p. 5, 2019.

[18] S. G. Vrachimis, D. G. Eliades, R. Taormina, A. Ostfeld, Z. Kapelan, S. Liu, M. Kyriakou, P. Pavlou, M. Qiu, and M. M. Polycarpou, "Battledim: Battle of the leakage detection and isolation methods," in *Proc., 2nd Int. CCWI/WDSA Joint Conf*, pp. 1–6, 2020.

[19] L. Testa, "B2-generation-for-cell-complexes," 2024. GitHub repository, accessed on March 13, 2025.