Complex-Valued Variational Autoencoders for Radar Detection in Joint Compound Gaussian Clutter and Thermal Noise

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***Abstract*—This paper introduces a novel Complex Variational AutoEncoder architecture leveraging complex-valued representations with a single latent channel for radar target detection. Unlike conventional real-valued VAEs, which require two latent channels to encode complex-valued data, the proposed approach directly operates in the complex domain, preserving the inherent structure of radar signals. By maintaining phase information and more accurately modeling the underlying data distribution, the complex-valued VAE enhances the separation of latent representations, leading to improved target discrimination.**

Experimental results demonstrate the effectiveness of the proposed CVAE in various clutter and noise scenarios against traditional radar detectors, such as the Matched Filter and Adaptive Normalized Matched Filter, and real-valued VAE.

***Index Terms*—Variational AutoEncoder, complex-valued neural networks, radar target detection**

I. Introduction

Detecting radar targets is a key challenge in signal processing [1], requiring accurate detection of targets amidst complex and heterogeneous background noise. Classical approaches, including the Matched Filter (MF), Normalized Matched Filter (NMF), and adaptive techniques like AMF [2], Kelly [3], and ANMF [4], are highly effective under Gaussian noise assumptions. However, their performance declines considerably in practical scenarios, where clutter exhibits more intricate statistical behaviors.

Recent progress in Deep Learning [5] has opened new possibilities for improving radar detection. Variational AutoEncoders (VAEs) have emerged as powerful tools for Out-Of-Distribution (OOD) detection [6], [7], thanks to their ability to model data distributions [8]. While VAEs have been successfully applied to anomaly detection in various domains including acoustic signal processing [9], medical imaging [10], and fault diagnosis in high-voltage equipment [11]—their potential for radar-based target detection remains largely unexplored.

In this paper, we extend the use of VAEs for radar detection [12] by introducing a complex-valued VAE (CVAE). Unlike real-valued VAEs, which require two latent channels to encode complex radar data, the proposed model directly operates in the complex domain, preserving phase information and enhancing latent space separation. Recent works have demonstrated the benefits of complex-valued VAEs in generative modeling for speech enhancement and object discovery [13]–[15]. In particular, complex recurrent VAEs improve temporal dependencies in speech resynthesis [13], while complex convolutional recurrent VAEs better capture phase-aware structures in noisy environments [14].

Beyond speech applications, complex-valued representations have been explored for unsupervised object discovery, demonstrating improved feature binding through phase alignment [16]. Additionally, contrastive learning techniques applied to complex-valued autoencoders have led to structured latent representations beneficial for segmentation and disentanglement tasks [15]. Moreover, complex-valued neural networks (CVNNs) have demonstrated both theoretical and practical advantages for radar signal processing, particularly in polarimetric SAR (PolSAR) classification, where they outperform real-valued networks even under capacity-equivalent constraints. This superiority is attributed to their ability to better exploit phase information, which is crucial for improving classification accuracy in complex radar data [17].

Building on these advances, we propose a CVAE tailored for radar target detection, leveraging complex-valued latent spaces to better handle clutter and improve robustness in adverse conditions. The paper is structured as follows: Section II reviews the statistical models and classical detectors commonly used in radar target detection. Section III details the proposed CVAE architecture and detection strategy. Section IV presents training and evaluation results, comparing the CVAE against Real-valued VAE and traditional detectors under various noise conditions. Finally, Section V concludes the paper, discussing the advantages of CVAEs in radar detection and future research directions.

Notations: Matrices are in bold and capital, vectors in bold. For any matrix $\mathbf{A}$, $\mathbf{A}^T$ is the transpose of $\mathbf{A}$ and $\mathbf{A}^H$ is the Hermitian transpose of $\mathbf{A}$. $\mathbf{I}$ is the identity matrix. $\mathcal{N}(\boldsymbol{\mu}, \mathbf{\Gamma})$ and $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{\Gamma})$ are respectively real and complex circular Normal distribution of mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Gamma}$. The matrix operator $\boldsymbol{\mathcal{T}}(.)$ is the Toeplitz matrix operator $\rho \rightarrow \{\boldsymbol{\mathcal{T}}(\rho)\}_{i,j} = \rho^{|i-j|}$. The symbols $\odot$ and $\oslash$ denotes the Hadamard element-wise product and division between vectors, respectively. The operators $|.|^{\circ}$, $.^{\circ\alpha}$ and $\log^{\circ}$ represent the Hadamard element-wise absolute value, power to α and

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logarithm of a vector, respectively. The symbols Re and Im denote the real and imaginary parts of a complex number.

II. Classical Radar Detection Methods

A. Hypothesis Testing and Signal Model

In adaptive radar detection, the objective is to identify the presence of a complex signal $\alpha\mathbf{p} \in \mathbb{C}^m$ within a received observation $\mathbf{z}$, which is contaminated by clutter noise $\mathbf{c}$ and an additive white Gaussian noise component $\mathbf{n}$, characterized by a covariance matrix $\sigma^2\mathbf{I}$ and independent from $\mathbf{c}$. In the case of a point-like target, this detection problem is formulated as a binary hypothesis test: $\begin{cases} H_0 : \mathbf{z} = \mathbf{c} + \mathbf{n}, \\ H_1 : \mathbf{z} = \alpha\,\mathbf{p} + \mathbf{c} + \mathbf{n}, \end{cases}$ where $\mathbf{z}$ represents the received signal, α is an unknown complex-valued target amplitude, and $\mathbf{p}$ is the known steering vector. In a homogeneous clutter setting, $\mathbf{c}$ follows a circular complex Gaussian distribution $\mathbb{C}\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_c)$. In a heterogeneous or partially homogeneous environment, a compound Gaussian model is used, where $\mathbf{c} = \sqrt{\tau}\mathbf{g}$, with $\mathbf{g} \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \tau\boldsymbol{\Sigma}_c)$, conditioned on a texture parameter $\tau \in \mathbb{R}^+$. The parameter τ captures variations in power across different radar cells and is assumed to have an expectation of $\mathbb{E}[\tau] = 1$ for simplicity. The Signal-to-Noise Ratio (SNR) under hypothesis H_1, after applying a whitening transformation, is given by $\text{SNR} = |\alpha|^2\,\mathbf{p}^H\,\boldsymbol{\Sigma}^{-1}\,\mathbf{p}$, where $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_c + \sigma^2\,\mathbf{I}$ where $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_c + \sigma^2\mathbf{I}$. Throughout this work, the power ratio $r = \mathrm{Tr}(\boldsymbol{\Sigma}_c)/(m\sigma^2)$ between clutter and thermal noise is fixed to 1.

In a thermal noise-free environment, optimal detection strategies exist for both the homogeneous and partially homogeneous cases, along with their adaptive counterparts. When the noise is Gaussian and the environment is homogeneous, the classical Matched Filter (MF) provides the optimal test:

$$\Lambda_{MF}(\mathbf{z}) = \frac{|\mathbf{p}^H\boldsymbol{\Sigma}^{-1}\mathbf{z}|^2}{\mathbf{p}^H\boldsymbol{\Sigma}^{-1}\mathbf{p}} \gtrless \lambda\,. \tag{1}$$

When both clutter and thermal noise share an unknown common covariance up to a scale factor, the scenario is classified as *partially homogeneous noise*. In such cases, the Normalized Matched Filter (NMF), which remains invariant to this unknown scale factor, is a suitable detector [4]:

$$\Lambda_{NMF}(\mathbf{z}) = \frac{\left|\mathbf{p}^H\boldsymbol{\Sigma}^{-1}\mathbf{z}\right|^2}{\left(\mathbf{p}^H\boldsymbol{\Sigma}^{-1}\mathbf{p}\right)\left(\mathbf{z}^H\boldsymbol{\Sigma}^{-1}\mathbf{z}\right)} \gtrless \lambda\,. \tag{2}$$

In scenarios where the covariance matrix is unknown but the noise remains Gaussian, adaptive two-step detectors are often used. These include the Adaptive Matched Filter (AMF-SCM) [2] and the Adaptive Normalized Matched Filter (ANMF-SCM) [18], where the true covariance matrix $\boldsymbol{\Sigma}$ is replaced by the Sample Covariance Matrix (SCM), estimated from secondary independent samples:

$$\widehat{\boldsymbol{\Sigma}}_{SCM} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{z}_k\,\mathbf{z}_k^H\,, \tag{3}$$

where $\mathbf{z}_k$ are clutter-only observations. When the clutter follows a compound Gaussian model ($\mathbf{c}_k = \sqrt{\tau_k}\mathbf{g}_k$), the false alarm regulation and detection performance of AMF-SCM and ANMF-SCM degrade significantly. A widely used alternative is the Tyler Adaptive Normalized Matched Filter (ANMF-FP), where the covariance matrix in (2) is estimated using the robust Tyler estimator built on secondary data $\mathbf{z}_k$'s [19], [20]:

$$\widehat{\boldsymbol{\Sigma}}_{FP} = \frac{m}{K}\sum_{k=1}^{K} \frac{\mathbf{z}_k^H\,\mathbf{z}_k}{\mathbf{z}_k^H\,\widehat{\boldsymbol{\Sigma}}_{FP}^{-1}\,\mathbf{z}_k}\,. \tag{4}$$

The NMF and ANMF-FP are particularly effective in scenarios with impulsive clutter, as they ensure robustness and texture invariance under H_0 conditions, unlike the Gaussian-based detectors (MF, AMF-SCM, ANMF-SCM), which fail in such cases [21], [22]. However, these detectors can suffer when additive thermal noise is present, as this affects the texture invariance of the Tyler estimator, impairing constant false alarm rate (CFAR) regulation. In these conditions, optimal detection schemes remain an open research problem. Importantly, when the clutter follows a Gaussian plus thermal noise model, classical detectors such as MF, AMF-SCM, and ANMF-SCM remain valid, as the sum of Gaussian-distributed noise components is still Gaussian. However, when clutter deviates from the Gaussian assumption, the limitations of these methods become evident. To tackle these challenges, we introduce a CVAE detector, leveraging the strengths of deep learning and complex-valued representations to effectively model cluttered radar environments with additive thermal noise which improves the latent feature representation and target discrimination.

III. From Real-Valued to Complex-Valued Variational AutoEncoders for Radar Signal Processing

To overcome the statistical model of clutter plus thermal noise, Variational AutoEncoders (VAEs) are powerful generative models that have demonstrated significant potential in radar target detection and anomaly detection. By learning a probabilistic latent space representation of the data, VAEs enable efficient encoding, reconstruction, and synthesis of radar signals, making them particularly well-suited for modeling complex data distributions.

To address these limitations, we propose a transition from real-valued VAEs to complex-valued VAEs, allowing direct processing of complex radar signals within a single latent channel. This approach not only maintains the integrity of the phase information but also enhances the separation of data in the latent space, and in term of reconstruction leading to improved target detection performance. In this section, we introduce our complex-valued VAE architecture and its advantages for radar applications.

A. Proposed Complex-Valued VAE Architecture

Our proposed VAE architecture is specifically designed to process and reconstruct complex radar signals while preserving their intrinsic structure. Unlike conventional real-valued VAEs, which require two separate latent channels to encode real and imaginary components, our model operates directly in the

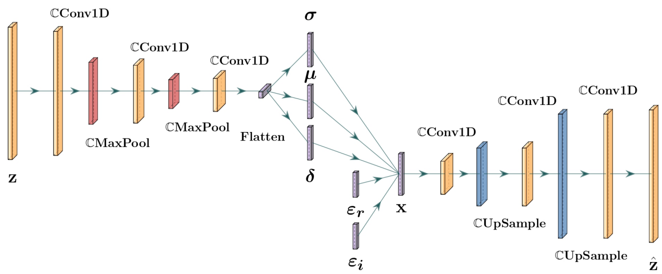


Fig. 1: Complex-Valued VAE network architecture

complex domain, thereby maintaining the phase information crucial for radar signal processing.

The architecture consists of an encoder and a decoder, each composed of complex convolutional layers. These layers are adapted to handle complex-valued inputs, enabling efficient feature extraction while maintaining the consistency of amplitude and phase relationships throughout the network.

1) Complex-valued Encoder: The encoder maps the high-dimensional complex radar signals into a lower-dimensional latent representation through a series of complex-valued convolutional layers. Each layer includes complex batch normalization and non-linear complex activation functions such as the complex-valued ReLU ($\mathbb{C}$ReLU). The downsampling process is performed via complex-valued pooling layers which select the complex-valued components that have the largest magnitude, ensuring that both magnitude and phase information are preserved during feature extraction.

At the final stage, the output is passed through fully connected layers that generate:

- The mean vector $\boldsymbol{\mu} \in \mathbb{C}^q$,
- The variance vector $\boldsymbol{\sigma}^{\circ 2} \in \mathbb{R}^{q+}$,
- The pseudo-variance term $\boldsymbol{\delta} \in \mathbb{C}^q$, which allows greater flexibility in modeling the latent space.

2) Complex Reparameterization Trick: To enable efficient sampling in the complex latent space while maintaining differentiability, we introduce a novel reparameterization trick. Given the estimated parameters $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\delta}$, the latent variable $\mathbf{z}$ is sampled as follows:

$$\mathbf{z} = \boldsymbol{\mu} + \boldsymbol{k}_r \odot \boldsymbol{\epsilon}_r + i\, \boldsymbol{k}_i \odot \boldsymbol{\epsilon}_i, \tag{5}$$

where: $$\begin{cases} \boldsymbol{k}_r = \frac{1}{\sqrt{2}} (\boldsymbol{\sigma} + \boldsymbol{\delta}) \oslash (\boldsymbol{\sigma} + \mathrm{Re}(\boldsymbol{\delta}))^{\circ \frac{1}{2}} , \\ \boldsymbol{k}_i = \frac{1}{2} \left(\boldsymbol{\sigma}^{\circ 2} - (|\boldsymbol{\delta}|^{\circ})^{\circ 2} \right)^{\circ \frac{1}{2}} \oslash (\boldsymbol{\sigma} + \mathrm{Re}(\boldsymbol{\delta}))^{\circ \frac{1}{2}} \end{cases} .$$

Here, $\boldsymbol{\epsilon}_r$ and $\boldsymbol{\epsilon}_i$ are identically and independently distributed according to standard Gaussian noise vectors $\mathcal{N}(\mathbf{0}, \mathbf{I})$.

To ensure numerical stability and enforce the constraint $|\boldsymbol{\delta}|^{\circ} < \boldsymbol{\sigma}$, the pseudo-variance is reparameterized as:

$$\boldsymbol{\delta} = \boldsymbol{\alpha} \odot \boldsymbol{\sigma} , \tag{6}$$

where each component of the q-vector $|\boldsymbol{\alpha}|^{\circ}$ is less than 1. This constraint is set using:

$$\boldsymbol{\alpha}_r = \mathrm{Re}(\boldsymbol{\delta}) \oslash (1 + |\boldsymbol{\delta}|^{\circ}) , \quad \boldsymbol{\alpha}_i = \mathrm{Im}(\boldsymbol{\delta}) \oslash (1 + |\boldsymbol{\delta}|^{\circ}) \ , \tag{7}$$

ensuring that the magnitude of $\boldsymbol{\delta}$ remains bounded.

3) Complex Decoder: The decoder reverses the encoding process by reconstructing the radar signal from the latent representation $\mathbf{z}$. It employs transposed complex convolutional layers combined with complex activation functions to progressively upsample and restore the original radar signal. The final output layer applies a complex convolution operation to generate the reconstructed complex-valued signal $\hat{\mathbf{z}} \in \mathbb{C}^N$.

4) Modified Complex VAE Loss Function: The VAE is trained by minimizing the evidence lower bound (ELBO), which is modified to account for the complex nature of the latent space. The total loss function is defined as:

$$\mathcal{L}_{\text{VAE}} = \mathcal{L}_{\text{rec}} + \beta\, \mathcal{D}_{\text{KL}} , \tag{8}$$

where the reconstruction loss $\mathcal{L}_{\text{rec}}$ is computed as:

$$\mathcal{L}_{\text{rec}} = \|\mathbf{z} - \hat{\mathbf{z}}\|^2 . \tag{9}$$

and where the KL divergence loss $\mathcal{D}_{\text{KL}}$ is adapted for the complex latent space:

$$\mathcal{D}_{\text{KL}} = \|\boldsymbol{\mu}\|^2 + \mathbf{1}_q^T \Big(\boldsymbol{\sigma} - \frac{1}{2} \log^{\circ} \left(\boldsymbol{\sigma}^{\circ 2} - \left(|\boldsymbol{\delta}|^{\circ} \right)^{\circ 2} \right) \Big). \tag{10}$$

The hyperparameter $\beta \in \mathbb{R}^+$ regulates the trade-off between reconstruction accuracy and latent space regularization.

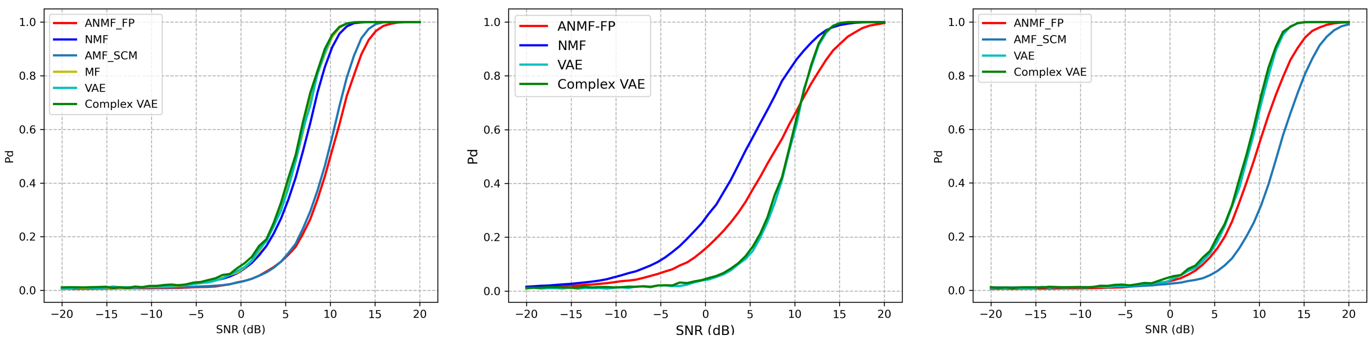
This complex-valued VAE architecture ensures efficient representation learning while preserving the fundamental properties of radar signals. The introduction of the pseudo-variance term $\boldsymbol{\delta}$ allows for finer control over the latent distribution, enhancing the model's ability to disentangle meaningful radar features.

B. Detection Strategy and False Alarm Regulation

During inference, the VAE is tasked with analyzing radar signals that may contain targets. Since it has been trained exclusively on clutter and noise, it struggles to accurately reconstruct inputs containing targets, leading to an increased Mean Squared Error (MSE) in such cases. Consequently, the detection criterion is established as $\mathcal{L}_{\text{rec}}(\mathbf{z}, \hat{\mathbf{z}}) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda_{\text{VAE}}$, where λ_{VAE} is a threshold determined from a validation dataset composed solely of clutter and noise, distinct from the training set. This threshold is set to maintain a predefined Probability of False Alarm (PFA) [23], ensuring system reliability in varying noise conditions. By leveraging its ability to learn intricate noise distributions, the CVAE provides a robust detection strategy while preserving strict false alarm regulation.

IV. Results Experimentations

This section evaluates the detection performance of the CVAE in comparison to classical radar detectors, namely AMF-SCM, ANMF-FP, and a RVAE. The experiments are conducted under different noise conditions, including correlated Compound Gaussian Noise (cCGN), correlated Gaussian Noise with Additive White Gaussian Noise (cGN + AWGN), and correlated Compound Gaussian Noise with Additive White Gaussian Noise (cCGN + AWGN). Performance is assessed in terms of Probability of Detection P_d as a function of SNR, with a fixed false alarm probability $P_{fa} = 10^{-2}$.



(a) cGN + AWGN (b) cCGN (c) cCGN + AWGN

Fig. 2: Detection performance under different noise for Doppler bin $d = 0$ ($P_{fa} = 10^{-2}$, $\rho = 0.5$, $\mu = 1$, $m = 16$, $K = 32$).

A. Signal and Noise Characteristics

The target echo is modeled as $\alpha = \sqrt{\text{SNR}}\, e^{2j\pi\phi}/\sqrt{m}$, where $\phi \in [0, 1]$, and the steering vector is given by $\mathbf{p} = \left(1, e^{2j\pi d \frac{1}{m}}, \ldots, e^{2j\pi d \frac{m-1}{m}}\right)^T$ for $m = 16$ bins, where d representing the target normalized Doppler bin index. The clutter covariance matrix follows $\boldsymbol{\Sigma}_c = \boldsymbol{\mathcal{T}}(\rho)$ with $\rho = 0.5$, while texture components τ and τ_k are sampled from a Gamma distribution $\Gamma(\mu, 1/\mu)$ with $\mu = 1$. For adaptive detectors, the covariance estimation uses SCM and Tyler estimators with $K = 2\,m$ independent secondary samples.

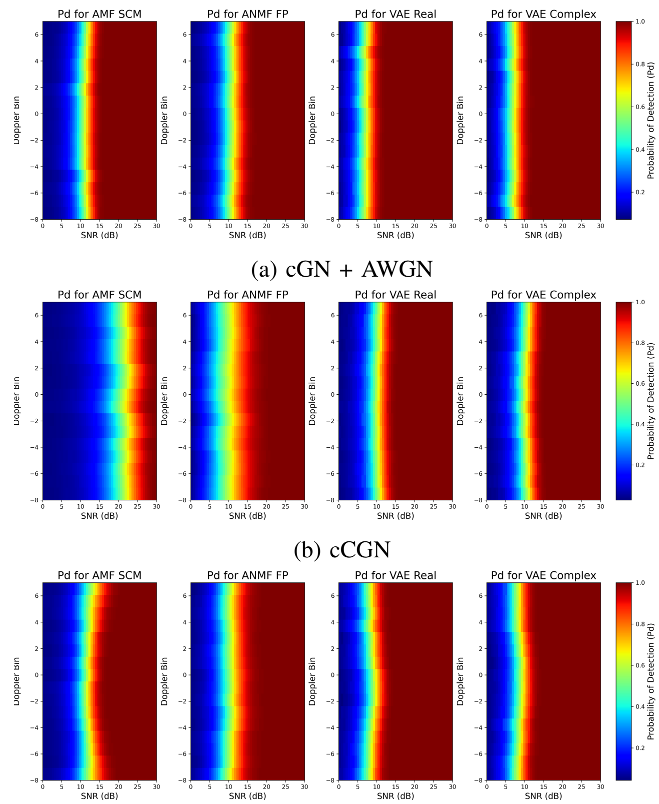
B. VAE Training Configuration

The VAE is trained on clutter plus noise Doppler profiles for each noise scenario. The dataset $\mathcal{D}_{H_0}$ consists of $15,000$ samples, two-thirds of which are allocated to training and the remaining third for validation. Training is performed over 50 epochs using the Adam optimizer [24] with a learning rate of 10^{-3}. The loss function $\mathcal{L}_{\text{VAE}}$ incorporates a regularization parameter of $\alpha = 10^2$, and the latent space dimension is set to 12. Once trained, detection is performed using the reconstruction loss $\mathcal{L}_{\text{rec}}$, with $P_{fa} = 10^{-2}$ determined from a validation set of $5,000$ independently generated samples.

C. Detection Performance for Zero Doppler Bin

Fig. 2 illustrates the detection results for Doppler bin $d = 0$ under various noise conditions.

In the cGN + AWGN scenario (Fig. 2-(a)), the complex-valued VAE exhibits similar performance to the real-valued VAE and the MF, demonstrating strong detection capabilities in Gaussian noise conditions. Under cCGN (Fig. 2-(b)), the complex-valued VAE remains competitive but lags behind the ANMF-FP at lower SNRs. However, as the SNR increases, its performance converges towards that of ANMF-FP, suggesting that while initial detection may be challenging, the model adapts well at higher SNR levels. In the cCGN + AWGN scenario (Fig. 2-(c)), the Complex VAE demonstrates superior robustness, surpassing ANMF-FP and AMF-SCM. This highlights its ability to effectively handle highly structured clutter environments with thermal noise contamination.



(a) cGN + AWGN

(b) cCGN

(c) cCGN + AWGN

Fig. 3: P_d-SNR-Doppler bin map comparing CVAE to RVAE, AMF and ANMF, under different noise scenarii.

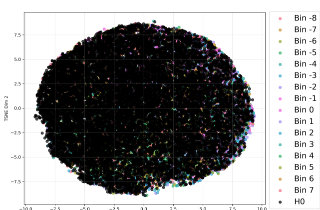
D. Detection Performance Across Doppler Bins

To provide a comprehensive evaluation, we analyze detection performance across all Doppler bins.

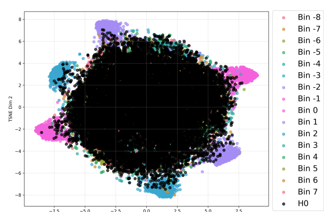
In cGN + AWGN (Fig. 3-(a)), both the Complex VAE and real-valued VAE consistently outperform adaptive detectors such as ANMF-FP and AMF-SCM, indicating resilience across varying Doppler conditions. For cCGN (Fig. 3-(b)), the Complex VAE performs similarly to the ANMF-FP at mid-to-high SNRs and surpasses AMF-SCM across all Doppler bins, confirming its robustness in non-Gaussian environments. Under cCGN + AWGN (Fig. 3-(c)), the Complex VAE outperforms all adaptive detectors, including ANMF-FP, across the entire Doppler spectrum, reinforcing its effectiveness in handling structured clutter and thermal noise mixtures.

E. Comparison Between Complex- and Real-Valued VAEs

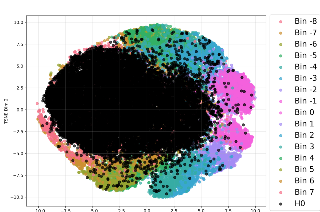
A key finding is that the CVAE and RVAE achieve comparable detection performance. However, the fundamental difference lies in the latent space representation: the CVAE, by directly operating in the complex domain, enables better separation between target and clutter distributions (see Figure 4). In contrast, the RVAE, constrained by its two-channel encoding, does not provide the same level of separation in the latent space. These results indicate that while both VAEs are viable alternatives to classical detectors, the CVAE offers a more structured and discriminative latent representation, making it particularly suited for radar signal processing.



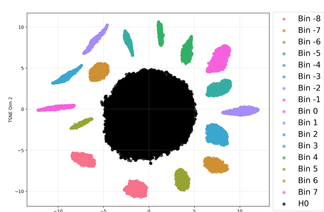
(a) RVAE SNR=10



(b) CVAE SNR=10



(c) RVAE SNR=20



(d) CVAE SNR=20

Fig. 4: Latent space separation CVAE vs RVAE

V. Conclusion

This work introduces a complex-valued Variational AutoEncoder for radar target detection, leveraging complex-valued representations to improve feature extraction in challenging noise environments. Comparative evaluations against AMF-SCM, ANMF-FP, and a real-valued VAE highlight that the Complex VAE achieves performance comparable to the real-valued VAE, with the primary difference being the structure of the learned latent space. These works demonstrate the potential of complex-valued deep learning for radar detection and suggest further exploration of hybrid real-complex architectures to optimize latent space interpretability and detection accuracy.

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