Event-Triggered Particle Filter with One-Step Randomly Delayed Measurement

Elhadi Gasmi*, Kundan Kumar*, Mohamed Amine Sid¶, and Simo Särkkä*, *Senior Member, IEEE*
*Department of Electrical Engineering and Automation, Aalto University, Finland
¶Mechatronics Laboratory (LMETR) - Optics and Precision Mechanics, Institute Ferhat Abbas University Setif 1, Algeria

***Abstract*—In this paper, we develop an event-triggered remote state estimator for nonlinear state-space models under network-induced one-step randomly delayed measurements. Adopting event-triggering strategies reduces the transmission burden between the sensor and the estimator while maintaining the estimation accuracy. The developed method employs a particle filter to approximate the posterior distribution using particles and weights. In the non-triggering case, we use the constrained Bayesian estimation to compute the integrals associated with the posterior distribution. We evaluate the performance of the proposed algorithm using a simulated aircraft tracking problem. The results show that the proposed algorithm provides a comparable estimation accuracy to a particle filter without event triggering.**

***Index Terms*—Cyber-physical systems, remote state estimation, event triggering, delayed measurement, particle filter.**

I. Introduction

In recent years, event-triggered remote state estimation strategies have gained significant attention in cyber-physical systems for their ability to maintain desirable estimation accuracy despite limited communication resources [1], [2]. The energy constraints of the sensor network and the finite bandwidth of the communication channel restrict the continuous transmission of sensor measurements to the remote estimator. The adoption of an event-triggering mechanism aims to achieve a desirable compromise between communication rate and estimation performance. In such a mechanism, measurement transmission is controlled by an event-triggered scheduler, which transmits the measurement only when a certain triggering condition is met.

Various event-triggering approaches in state estimation have been explored [3]–[7]. In [3], Miskowicz introduced the open-loop send-on-delta (SOD) method, a simple yet widely adopted approach. The authors in [4] propose innovation-based triggering, where the triggering condition relies on the predicted state estimate from the linear remote estimator. In [6], the authors proposed an open-loop and a closed-loop stochastic event-triggering strategy for measurement transmission. For linear state space models, a variance-based triggering mechanism is developed in [5], where the authors discuss the switching Riccati equation for an event-triggered Kalman filter (KF). In [8], a sum of Gaussians approach to deal with the event-triggered measurements is proposed.

For nonlinear systems, an event-triggered extended Kalman filter (EEKF) is proposed in [9]. In [10], an event-triggered condition is realized for a nonlinear system with packet dropout conditions using an unscented Kalman filter (UKF). The same approach has been implemented using the cubature Kalman filter (CKF) in [11], [12]. However, these works are based on a Gaussian approximation for the estimator.

Corresponding author: E. Gasmi (email: elhadi.gasmi@aalto.fi).

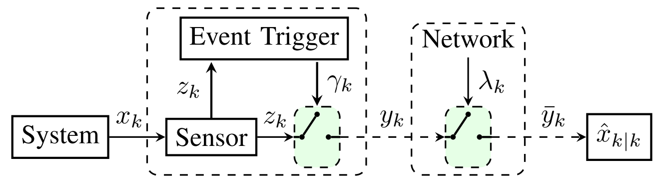


Fig. 1 Illustration of the concept of the paper, which combines an event-triggering strategy with randomly delayed measurements in cyber-physical systems.

For highly nonlinear systems, particle filters often provide better estimation accuracy at the cost of high computational burden [13], [14]. The event-trigger-based particle filters (EPF) are explored in [15]–[19]. The EPF with SOD triggering and innovation-based triggering are proposed in [18] and [19], respectively. Ruuskanen *et al.* [20] proposed an enhanced event-based auxiliary particle filtering approach for resource-constrained remote state estimation.

Due to the limitation of the communication channel, several network-induced phenomena may occur, such as random delay in measurement, packet dropout, and fading measurement, among others [21], [22]. Early work on filtering with delayed measurements can be found in [23], where the KF was modified to incorporate the delay. The KF is proposed for randomly sampled and delayed measurements in [24]. In [25], the CKF for one-step randomly delayed measurement is formulated, and the work is extended to account for correlated noise in [26]. A modified PF to deal with one-step randomly delayed measurements and unknown latency probability has been developed in [27]. However, there is a lack of event triggering-based particle filtering methods that account for random measurement delays. Our aim is to address this gap.

The main contribution of this paper is an event-triggered PF to handle one-step delayed measurements (EPF-OD). The method uses a constrained Bayesian estimation approach to compute the integrals that arise in the event-triggering model. Additionally, we numerically evaluate the performance of the proposed methods. The method is illustrated in Fig. 1.

II. Problem Formulation and Preliminaries

Consider the following nonlinear stochastic dynamic system in the discrete-time domain

$$x_{k+1} = f_k(x_k) + w_k, \tag{1a}$$

$$z_k = h_k(x_k) + v_k, \tag{1b}$$

where $x_k \in \mathbb{R}^n$ is the system state at time step k, $z_k \in \mathbb{R}^{n_z}$ is the sensor measurement, and $f_k : \mathbb{R}^n \to \mathbb{R}^n$ and $h_k : \mathbb{R}^n \to \mathbb{R}^{n_z}$ are known nonlinear dynamic and measurement functions, respectively. The process noise w_k, and the measurement noise v_k are uncorrelated white noises with probability density functions (PDFs) p_w and p_v, respectively.

This paper considers networks with constrained bandwidth, such as low-bandwidth wireless sensor networks. Thus, to reduce the communication burden, an event-triggered mechanism [28] is introduced, as depicted in Fig. 1. Let γ_k be the trigger-decision variable given as

$$\gamma_k = \begin{cases} 0, & z_k \in \Xi_k(\bar{z}), \\ 1, & \text{otherwise}, \end{cases} \tag{2}$$

where $\Xi_k(\bar{z})$ is the non-trigger set according to the SOD strategy defined as $\{z_k \in \mathbb{R}^{n_z} \mid (z_k - \bar{z})^T(z_k - \bar{z}) \leq \xi\}$ with $\bar{z}$ being the last transmitted measurement, and $\xi \geq 0$ the fixed predefined threshold. If $\gamma_k = 1$, the sensor sends the measurement z_k to the remote estimator which we denote as $\bar{y}_k = z_k$; otherwise, no measurement is transmitted. In the non-trigger case, we assume that the remote estimator knows that the measurement z_k lies in the set $\Xi_k(\bar{z})$. Therefore, the measurement likelihood at the remote estimator, $p(\bar{y}_k \mid x_k)$, in the non-triggering case can be expressed as [20]

$$\begin{aligned} p(\bar{y}_k \mid x_k) &\propto \int \mathcal{U}(z_k \in \Xi_k(\bar{z})) p(z_k \mid x_k) dz_k, \\ &\propto \int_{z_k \in \Xi_k(\bar{z})} p(z_k \mid x_k) dz_k, \end{aligned} \tag{3}$$

where $\mathcal{U}(z_k \in \Xi_k(\bar{z}))$ is the uniform distribution of z_k over the non trigger set $\Xi_k(\bar{z})$.

As shown in Fig. 1, the sensor measurement z_k is transmitted to the remote estimator through an unreliable communication channel, which may introduce a one-step delay in the measurement [27]. To model the delay in the measurement, we introduce a Bernoulli random variable $\lambda_k \in \{0, 1\}$ with parameter α, which consequently has the following properties: $p\{\lambda_k = 1\} = \mathbb{E}\{\lambda_k\} = \alpha$ and $p\{\lambda_k = 0\} = 1 - \mathbb{E}\{\lambda_k\} = 1 - \alpha$. We write the measurement model Eq. (1b) using the variable (λ_k) to account for one step delay measurement, as follows [27]:

$$y_k = (1 - \lambda_k) z_k + \lambda_k z_{k-1}. \tag{4}$$

The value of the trigger variable (γ_k) is assumed to be known to the remote estimator [4], [28]. Using Eq. (2) and (4), we can reformulate the measurement model at the remote estimator as follows:

$$\bar{y}_k = \begin{cases} (1 - \lambda_k) z_k + \lambda_k z_{k-1}, & \gamma_k = 1, \\ \{z_k \in \Xi_k(\bar{z})\}, & \gamma_k = 0. \end{cases} \tag{5}$$

This paper aims to develop an event-trigger-based remote state estimation algorithm for nonlinear stochastic systems with one-step delayed measurement. Let $\bar{y}_{1:k} = \{\{\bar{y}_1, \gamma_1\}, \{\bar{y}_2, \gamma_2\}, \ldots, \{\bar{y}_k, \gamma_k\}\}$ represent the information set available to the remote estimator. We aim to compute the approximate posterior distribution of the state $p(x_k \mid \bar{y}_{1:k})$ using particle filtering.

III. Event-Triggered Particle Filter with One Step Delayed Measurement

In this section, we formulate the EPF-OD based on the particle filter approximation. In particle filtering, the posterior distribution of state $p(x_k \mid \bar{y}_{1:k})$ is approximated using the sequential importance resampling (SIR) method, which relies on the generated particles and their corresponding weights, $\{x_k^{(i)}, w_k^{(i)}\}_{i=1}^M$ [15]–[17], as follows:

$$p(x_k \mid \bar{y}_{1:k}) \approx \hat{p}(x_k \mid \bar{y}_{1:k}) = \frac{1}{M} \sum_{i=1}^{M} \delta(x_k - x_k^{(i)}), \tag{6}$$

where M is the number of particles and $\delta(\cdot)$ is the Dirac delta function. However, due to the event triggering mechanism, the measurements z_k are not available to the remote estimator at all time instants k. Thus, approximating $p(x_k \mid \bar{y}_{1:k})$ requires analyzing two cases: when a measurement is transmitted, $\gamma_k = 1$, and when no measurement is transmitted, $\gamma_k = 0$.

A. Non-triggering case ($\gamma_k = 0$)

When the measurement is not transmitted, the information set available at the remote estimator is $\bar{y}_{1:k}$, where $\bar{y}_k = \{z_k \in \Xi_k(\bar{z})\}$. Consequently, the posterior PDF is given by the Bayes' rule

$$p(x_k \mid \bar{y}_{1:k}) \propto \int_{\Xi_k(\bar{z})} p(z_k \mid x_k) dz_k \, p(x_k \mid \bar{y}_{1:k-1}), \tag{7}$$

where $p(x_k \mid \bar{y}_{1:k-1})$ is the prior PDF obtained from the prediction step

$$p(x_k \mid \bar{y}_{1:k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid \bar{y}_{1:k-1}) \, dx_{k-1}, \tag{8}$$

with $p(x_{k-1} \mid \bar{y}_{1:k-1})$ being the posterior PDF at $k-1$. In PF, $p(x_{k-1} \mid \bar{y}_{1:k-1})$ is approximated using particles, such that

$$p(x_{k-1} \mid \bar{y}_{1:k-1}) \approx \frac{1}{M} \sum_{i=1}^{M} \delta(x_{k-1} - x_{k-1}^{(i)}), \tag{9}$$

where $x_{k-1}^{(i)}$ is the (resampled) i-th particle at time step $k-1$.

Let us assume that the particles $\{x_k^{(i)}\}_{i=1}^M$ are generated by using the dynamic model $p(x_k \mid x_{k-1}^{(i)})$ as the importance distribution. Hence, we can generate samples from the distribution via

$$x_k^{(i)} = f(x_{k-1}^{(i)}) + w_k^{(i)}, \quad \forall i = 1, 2, \ldots, M, \tag{10}$$

where the noise $w_k^{(i)}$ is sampled from $p_w(\cdot)$. The PF-approximated posterior distribution becomes

$$\hat{p}(x_k \mid \bar{y}_{1:k}) = \sum_{i=1}^{M} \tilde{\omega}_k^{(i)} \delta(x_k - x_k^{(i)}), \tag{11}$$

where $\tilde{\omega}_k^{(i)}$ is the weight corresponding to i-th particle, given by

$$\tilde{\omega}_k^{(i)} \propto \int_{z_k \in \Xi_k(\bar{z})} p(z_k \mid x_k^{(i)}) dz_k. \tag{12}$$

Please note that computing the weights requires solving the integral Eq. (12). Obtaining a solution for the integral is a challenging task in real time. To overcome this problem, we use constrained Bayesian state estimation [28], [29]. We define a constraint set Ω_k as the non trigger set, $\Xi_k(\bar{z})$, and the constraint function, $\phi_k(x_k)$ as the measurement z_k. So, the particle constraint function $\phi_k^{(i,j)}(x_k)$ is defined as the particle measurement $z_k^{(i,j)}$ for each particle $x_k^{(i)}$, given by

$$\phi_k^{(i,j)}(x_k) = h_k(x_k^{(i)}) + v_k^{(i,j)} = z_k^{(i,j)}. \tag{13}$$

If all the particle measurements $\{z_k^{(i)}\}_{i=1}^M$ are generated according to Eq. (13), the PDF $p(z_k \mid x_k^{(i)})$ in Eq. (12) can be approximated as

$$p(z_k \mid x_k^{(i)}) \approx \hat{p}(z_k \mid x_k^{(i)}) = \frac{1}{\tilde{M}} \sum_{j=1}^{\tilde{M}} \delta(z_k - z_k^{(i,j)}). \tag{14}$$

Let $L_{\Xi_k(\bar{z})}^{(i,j)}(x_k^{(i,j)}, v_k^{(i,j)})$ be the corresponding judgment variable that takes 1 if the particle measurement $z_k^{(i,j)}$ from of the set $\{z_k^{(i,j)}\}_{j=1}^{\tilde{M}}$ satisfies $z_k^{(i,j)} \in \Xi_k(\bar{z})$. Then the particle weights in Eq. (12) can be approximated as

$$\tilde{\omega}_k^{(i)} \propto \frac{1}{\tilde{M}} \sum_{j=1}^{\tilde{M}} L_{\Xi_k(\bar{z})}^{(i,j)}(x_k^{(i,j)}, v_k^{(i,j)}). \tag{15}$$

B. Triggering case ($\gamma_k = 1$)

When the sensor measurement z_k is transmitted to the remote estimator, a delay may occur. To model that, using Eq. (5), the likelihood function $p(\bar{y}_k \mid x_k, x_{k-1})$, which depends on both x_k and x_{k-1}, can be expressed as

$$\begin{aligned} p(\bar{y}_k \mid x_k, x_{k-1}) &= \sum_{\lambda_k} p(\bar{y}_k \mid \lambda_k, x_k, x_{k-1}) p(\lambda_k) \\ &= \alpha p(\bar{y}_k \mid x_k, x_{k-1}, \lambda_k = 1) + (1-\alpha) \\ &\quad \times p(\bar{y}_k \mid x_k, x_{k-1}, \lambda_k = 0), \end{aligned} \tag{16}$$

where λ_k is assumed to be independent of the state and event-triggered variable. The likelihood function can be written as

$$p(\bar{y}_k \mid x_k, x_{k-1}) = \alpha p(z_k \mid x_{k-1}) + (1-\alpha) p(z_k \mid x_k). \tag{17}$$

If particles $x_k^{(i)}$ are sampled from the importance distribution $p(x_k \mid x_{k-1}^{(i)})$, as in Eq. (10), the importance weights can be evaluated as

$$\tilde{\omega}_k^{(i)} \propto \alpha p(\bar{y}_k \mid x_{k-1}^{(i)}) + (1-\alpha) p(\bar{y}_k \mid x_k^{(i)}). \tag{18}$$

Therefore, using Eq. (15) and Eq. (18), the corresponding importance weights for trigger and non-trigger cases can be computed as

$$\tilde{\omega}_k^{(i)} \propto \begin{cases} \alpha p(\bar{y}_k \mid x_{k-1}^{(i)}) + (1-\alpha) p(\bar{y}_k \mid x_k^{(i)}), & \gamma_k = 1, \\ \frac{1}{\tilde{M}} \sum_{j=1}^{\tilde{M}} L_{\Xi_k(\bar{z})}^{(i,j)}(x_k^{(i,j)}, v_k^{(i,j)}), & \gamma_k = 0. \end{cases} \tag{19}$$

The pseudo-code for the proposed EPF-OD is provided in Algorithm 1.

Algorithm 1: Event trigger particle filter with one step delayed measurement (EPF-OD)

Initialization: for $i = 1, \ldots, M$, draw $x_0^{(i)} \sim p(x_0)$.
For time $k = 1, 2, \ldots, N$

1) Importance sampling: for $i = 1, \ldots, M$, sample particles $x_k^{(i)} \sim p(x_k \mid x_{k-1}^{(i)})$.
2) Weight update: for $i = 1, \ldots, M$, calculate the weight $\tilde{\omega}_k^{(i)}$ in Eq. (19).
3) Weight normalization: for $i = 1, \ldots, M$, normalize the weight $\omega_k^{(i)} = \tilde{\omega}_k^{(i)} / \sum_{i=1}^M \tilde{\omega}_k^{(i)}$.
4) State estimate: Compute the posterior state estimate and error covariance
   - $\hat{x}_{k|k} \approx \sum_{i=1}^M \omega_k^{(i)} x_k^{(i)}$
   - $P_{k|k} \approx \sum_{i=1}^M \omega_k^{(i)} (x_k^{(i)} - \hat{x}_{k|k})(x_k^{(i)} - \hat{x}_{k|k})^T$.
5) Resampling Step: Generate M new samples $x_k^{(j)}$ from particles $x_k^{(i)}$ according to the weights $\omega_k^{(i)}$.

Return results as $\hat{x}_{k|k}, \hat{P}_{k|k}, \{x_k^{(i)}\}_{i=1}^M$ for $k = 1, \ldots, N$.

IV. Simulation Results

In this section, we evaluate the performance of the proposed EPF-OD on a simulated aircraft tracking problem [30], where an aircraft maneuvers in a horizontal plane, with an unknown turn rate. The target dynamics model in discrete-time domain is given by:

$$x_{k+1} = \begin{pmatrix} 1 & \frac{\sin(\Omega_k T_{in})}{\Omega_k} & 0 & \frac{1-\cos(\Omega_k T_{in})}{\Omega_k} & 0 \\ 0 & \cos(\Omega_k T_{in}) & 0 & -\sin(\Omega_k T_{in}) & 0 \\ 0 & \frac{1-\cos(\Omega_k T_{in})}{\Omega_k} & 1 & \frac{\sin(w_k T_{in})}{\Omega_k} & 0 \\ 0 & \sin(\Omega_k T_{in}) & 0 & \cos(\Omega_k T_{in}) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} x_k + w_k,$$

where $x_k = [x_{1,k}, x_{2,k}, x_{3,k}, x_{4,k}, \Omega_k]$ is the target state, $x_{1,k}$ and $x_{3,k}$ are the position, $x_{2,k}$ and $x_{4,k}$ are the velocity, in x and y directions, respectively; Ω_k is the turn rate; the parameter T_{in} is the time-interval between two consecutive measurements; the process noise $w_k \sim \mathcal{N}(0, Q)$.

The sensor measures range and bearing, and is equipped with an event-triggering scheduler. The measurement model can be expressed as

$$z_k = \begin{pmatrix} \sqrt{x_{1,k}^2 + x_{3,k}^2} \\ \tan^{-1} \frac{x_{3,k}}{x_{1,k}} \end{pmatrix} + v_k,$$

where the measurement noise $v_k \sim \mathcal{N}(0, R)$. The associated parameters x_0, $\hat{x}_{0|0}$, $P_{0|0}$, T_{in}, Q, and R are consistent with those detailed in [30]. The target trajectory is simulated for 100 time steps.

In this problem, we implement the PF-OD [27] and the proposed EPF-OD. The number of particles used in both PF-OD and EPF-OD is $M = 10000$. We set the threshold value ξ to 8×10^4 and the latency probability for the one-step delayed measurement to $\alpha = 0.20$. Fig. 2a shows that both PF-OD and the proposed EPF-OD successfully track the true target trajectory. We compare the performance of the estimators in terms of position root mean square error (RMSE), evaluated over 100 Monte Carlo (MC) runs, plotted in Fig. 2b. The figure indicates that the proposed EPF-OD achieves a similar RMSE as PF-OD even with 68% of the communication rate.

Furthermore, we assess the performance of the estimators with varying α and ξ values in terms of position RMSE, as illustrated in Fig. 3 and Table I. Fig. 3a demonstrates that the position RMSE of EPF-OD increases slightly as α values rise. A similar trend is observed for the proposed EPF-OD across different ξ values in Fig. 3b. In Table I, we present the average position RMSE results for different values of ξ and α. From the table, we observe that the average RMSE of the proposed EPF-OD increases as α and ξ values increase.

Figure 4 presents the plot of the average communication rate Γ versus ξ, showing that Γ decreases as ξ increases. We plot the average RMSE of the proposed EPF-OD versus ξ with different α values in Fig. 5. The figure indicates that the higher α and ξ values lead to an increase in average RMSE. Table II presents the relative computational time of EPF-OD with respect to PF-OD, with fixed $\alpha = 0.5$ and varying ξ values. The results indicate that EPF-OD attains a slightly higher computation cost than PF-OD.

TABLE I Average position RMSE of PF-OD and EPF-OD for different ξ and α.

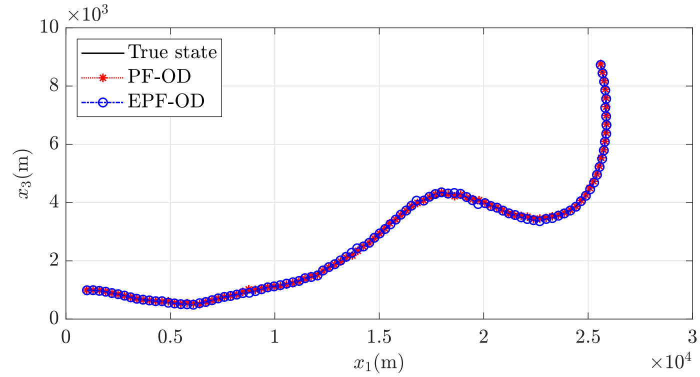
α	**Filter**	**Communication rate Γ (in %)** 90% ($\xi = 5e4$)	68% ($\xi = 8e4$)	48% ($\xi = 15e4$)
0.1	PF-OD	22.19	22.19	22.19
	EPF-OD	22.97	26.71	41.7
0.3	PF-OD	24.4	24.4	24.4
	EPF-OD	26.5	28.3	42.7
0.5	PF-OD	26.4	26.4	26.4
	EPF-OD	28.6	39.7	49.8

TABLE II Relative computation time of PF-OD and EPF-OD

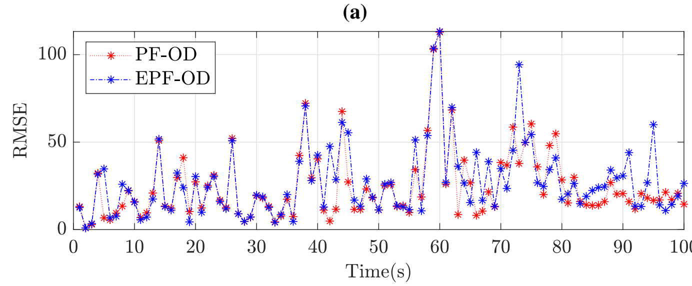
α	**Filter**	**Communication rate Γ (in %)** 90% ($\xi = 5e4$)	68% ($\xi = 8e4$)	48% ($\xi = 15e4$)
0.5	PF-OD	1	1	1
	EPF-OD	1.07	1.08	1.44

V. CONCLUSIONS

In this paper, we solve the event-triggered one-step delay estimation problem by employing a particle filter to approximate the filtering posterior distribution using particles and

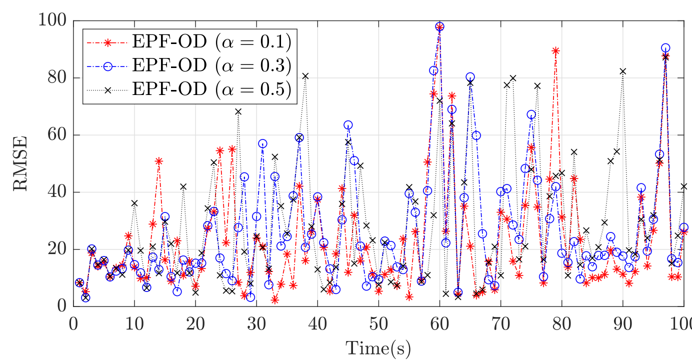


(a)

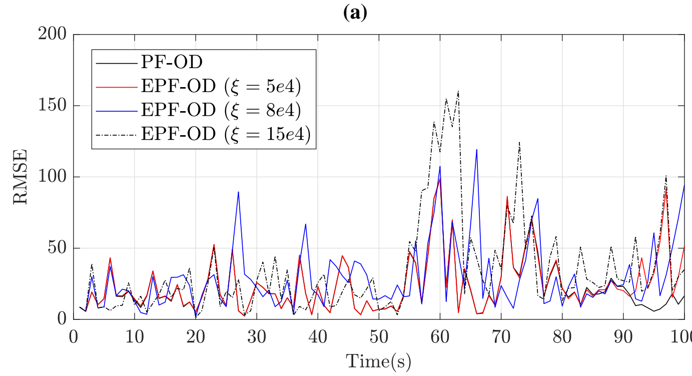


(b)

Fig. 2 (a) The true and estimated target trajectories and (b) RMSE in position for different estimators.



(a)



(b)

Fig. 3 The position RMSE for different (a) α values and (b) ξ values.

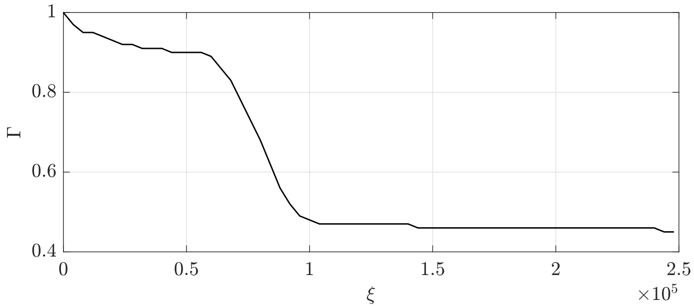


Fig. 4 Sensor communication rate Γ as function of ξ.

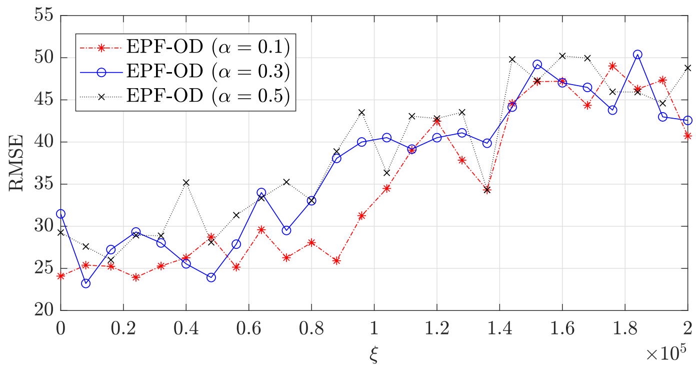


Fig. 5 Average position RMSE as function of ξ for different α values.

weights. The integrals arising in the non-triggered case are computed using constrained Bayesian estimation. Simulations are conducted on an aircraft tracking problem with varying latency probabilities and triggering thresholds. The results demonstrate the effectiveness of the proposed EPF-OD method under communication constraints.

REFERENCES

[1] L. Canzian and M. van der Schaar, "Timely event detection by networked learners," *IEEE Transactions on Signal Processing*, vol. 63, no. 5, pp. 1282–1296, 2015.

[2] D. Han, K. You, L. Xie, J. Wu, and L. Shi, "Optimal parameter estimation under controlled communication over sensor networks," *IEEE Transactions on Signal Processing*, vol. 63, no. 24, pp. 6473–6485, 2015.

[3] M. Miskowicz, "Send-on-delta concept: An event-based data reporting strategy," *Sensors*, vol. 6, pp. 49–63, 1 2006.

[4] J. Wu, Q. S. Jia, K. H. Johansson, and L. Shi, "Event-based sensor data scheduling: Trade-off between communication rate and estimation quality," *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 1041–1046, 2013.

[5] S. Trimpe and R. D'andrea, "Event-based state estimation with variance-based triggering," *IEEE Transactions on Automatic Control*, vol. 59, pp. 3266–3281, 2 2012.

[6] D. Han, Y. Mo, J. Wu, S. Weerakkody, B. Sinopoli, and L. Shi, "Stochastic event-triggered sensor schedule for remote state estimation," *IEEE Transactions on Automatic Control*, vol. 60, no. 10, pp. 2661–2675, 2015.

[7] J. W. Marck and J. Sijs, "Relevant sampling applied to event-based state-estimation," in *2010 Fourth International Conference on Sensor Technologies and Applications*, pp. 618–624, 2010.

[8] J. Sijs and M. Lazar, "Event based state estimation with time synchronous updates," *IEEE Transactions on Automatic Control*, vol. 57, no. 10, pp. 2650–2655, 2012.

[9] X. Zheng and H. Fang, "Recursive state estimation for discrete-time nonlinear systems with event-triggered data transmission, norm-bounded uncertainties and multiple missing measurements," *International Journal of Robust and Nonlinear Control*, vol. 26, no. 17, pp. 3673–3695, 2016.

[10] L. Li, D. Yu, H. Yang, and C. Yan, "UKF for nonlinear systems with event-triggered data transmission and packet dropout," in *2016 3rd International Conference on Informative and Cybernetics for Computational Social Systems (ICCSS)*, pp. 37–42, IEEE, 2016.

[11] M. Kooshkbaghi and H. J. Marquez, "Event-triggered discrete-time cubature Kalman filter for nonlinear dynamical systems with packet dropout," *IEEE Transactions on Automatic Control*, vol. 65, pp. 2278–2285, 5 2020.

[12] M. Kooshkbaghi, H. J. Marquez, and W. Xu, "Event-triggered approach to dynamic state estimation of a synchronous machine using cubature Kalman filter," *IEEE Transactions on Control Systems Technology*, vol. 28, no. 5, pp. 2013–2020, 2019.

[13] S. Särkkä and L. Svensson, *Bayesian Filtering and Smoothing*. Cambridge University Press, 2nd ed., 2023.

[14] N. Chopin and O. Papaspiliopoulos, *An Introduction to Sequential Monte Carlo*. Springer, 2020.

[15] M. G. Cea and G. C. Goodwin, "Event based sampling in non-linear filtering," *Control Engineering Practice*, vol. 20, no. 10, pp. 963–971, 2012.

[16] W. Li, Z. Wang, Q. Liu, and L. Guo, "An information aware event-triggered scheme for particle filter based remote state estimation," *Automatica*, vol. 103, pp. 151–158, 2019.

[17] S. Davar and A. Mohammadi, "Event-based particle filtering with point and set-valued measurements," *25th European Signal Processing Conference, EUSIPCO 2017*, pp. 211–215, 2017.

[18] M. A. Sid and S. Chitraganti, "Nonlinear event-based state estimation using particle filtering approach," *Proceedings of 2016 8th International Conference on Modelling, Identification and Control, ICMIC 2016*, pp. 874–879, 2017.

[19] S. Chitraganti and M. A. Darwish, "Nonlinear event-based state estimation using sequential Monte Carlo approach," in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, pp. 2170–2175, IEEE, 2017.

[20] J. Ruuskanen and A. Cervin, "Event-based state estimation using the auxiliary particle filter," in *2019 18th European Control Conference (ECC)*, pp. 1854–1860, IEEE, 2019.

[21] R. Caballero-Águila, A. Hermoso-Carazo, and J. Linares-Pérez, "Linear estimation based on covariances for networked systems featuring sensor correlated random delays," *International Journal of Systems Science*, vol. 44, no. 7, pp. 1233–1244, 2013.

[22] J. Hu, Z. Wang, B. Shen, and H. Gao, "Gain-constrained recursive filtering with stochastic nonlinearities and probabilistic sensor delays," *IEEE Transactions on Signal Processing*, vol. 61, no. 5, pp. 1230–1238, 2012.

[23] A. Ray, L. W. Lion, and J. H. Shen, "State estimation using randomly delayed measurements," *Journal of Dynamic Systems, Measurement, and Control*, vol. 115, no. 1, pp. 19–26, 1993.

[24] S. C. Thomopoulos and L. Zhang, "Decentralized filtering with random sampling and delay," *Information Sciences*, vol. 81, no. 1-2, pp. 117–131, 1994.

[25] X. Wang, Y. Liang, Q. Pan, and C. Zhao, "Gaussian filter for nonlinear systems with one-step randomly delayed measurements," *Automatica*, vol. 49, no. 4, pp. 976–986, 2013.

[26] X. Wang, Y. Liang, Q. Pan, C. Zhao, and F. Yang, "Design and implementation of Gaussian filter for nonlinear system with randomly delayed measurements and correlated noises," *Applied Mathematics and Computation*, vol. 232, pp. 1011–1024, 2014.

[27] Y. Zhang, Y. Huang, N. Li, and L. Zhao, "Particle filter with one-step randomly delayed measurements and unknown latency probability," *International Journal of Systems Science*, vol. 47, no. 1, pp. 209–221, 2016.

[28] X. Liu, L. Li, Z. Li, X. Chen, T. Fernando, H. H. C. Iu, and G. He, "Event-trigger particle filter for smart grids with limited communication bandwidth infrastructure," *IEEE Transactions on Smart Grid*, vol. 9, no. 6, pp. 6918–6928, 2018.

[29] S. Li, L. Li, Z. Li, X. Chen, T. Fernando, H. H. C. Iu, G. He, Q. Wang, and X. Liu, "Event-trigger heterogeneous nonlinear filter for wide-area measurement systems in power grid," *IEEE Transactions on Smart Grid*, vol. 10, no. 3, pp. 2752–2764, 2019.

[30] I. Arasaratnam and S. Haykin, "Cubature Kalman filters," *IEEE Transactions on Automatic Control*, vol. 54, no. 6, pp. 1254–1269, 2009.