A Zoom Algorithm for Spectral Correlation Density Estimation

Antonio Napolitano
Department of Engineering
University of Napoli "Parthenope"
Napoli, Italy
antonio.napolitano@uniparthenope.it

***Abstract*—A new spectral correlation density (SCD) estimator is proposed. It exploits the chirp zeta transform algorithm in a modified version of the strip spectral correlation analyzer (SSCA) and is referred to as zoom SSCA. It provides an estimate of the SCD in the desired cycle- and spectral-frequency rectangular region without computing the SCD in the whole domain. The cycle- and spectral-frequency analysis intervals can be independently selected and the cycle- and spectral-frequency resolution is constant in the whole analysis region. As an example of application, the cyclic spectral analysis of a covert weak communication signal in the presence of strong noise and interference is carried out.**

***Index Terms*—cyclostationarity, spectral correlation, cyclic spectrum estimation, strip spectral correlation analyzer**

I. Introduction

Spectral correlation density (SCD) or cyclic spectrum estimation is exploited in cyclostationarity-based signal processing algorithms in communications, radar/sonar, circuits and control, acoustics and mechanics, econometrics, climatology, biology, and astrophysics [9, Chap. 10].

These algorithms can be suitably implemented due to the existence of consistent estimators of the cyclic statistical functions [7, Chap. 11], [9, Chap. 5]. In particular, for the SCD estimation, the time- and frequency-smoothed cyclic periodograms have been proposed. Efficient implementations of these estimators are the fast Fourier transform (FFT) accumulation method (FAM) and the strip spectral correlation analyzer (SSCA) [4]. Parallel architectures for FAM and SSCA algorithms are presented in [5], [11].

Fast or low-complexity algorithms for computing the SCD in the whole cycle- and spectral-frequency domain have also been presented in [8], [12], [13], [14] in the context of radio-frequency spectrum sensing. In [1] and [3], in the context of mechanical engineering, estimators have been presented with some constraints on the maximum observable cycle frequency.

In electromagnetic-spectrum monitoring in a non-cooperative scenario, in order to discover covert communication signals [2], [6] in the presence of strong noise and interfering signals, an estimate of the SCD must be performed in appropriate intervals of cycle frequencies α and spectral frequencies ν where significant cyclic features of the covert signal are expected to be found.

In the present paper, a versatile and memory-parsimonious SCD analyzer is proposed, which is referred to as zoom strip spectral correlation analyzer (ZSSCA), aimed at estimating the SCD in a desired rectangular region of the (α, ν) domain. The chirp zeta transform (CZT) [10] is exploited in a modified version of the originally proposed SSCA [4] to zoom in the cycle-frequency domain. The modification here proposed consists in considering strips in the (α, ν) domain that are parallel to the spectral-frequency axis rather than to diagonal lines. Due to such a modification, the cycle-frequency and spectral-frequency search intervals can be chosen independently, without any constraint, and the cycle-frequency resolution is constant for all the points on the grid on the analysis rectangular region. The zoom in the spectral-frequency domain is obtained by channelization, that is, by passing the input signal through band-pass filters whose bandwidth is equal to the desired spectral-frequency resolution. The central frequency of the filters ranges in the spectral-frequency analysis interval with a spacing equal to the spectral-frequency resolution.

The ZSSCA acts as a magnifying glass in the desired region of the (α, ν) domain without requiring the estimation of the SCD in the whole domain, with an advantage in terms of computational and memory costs. The advantage is significant when large data-record lengths are required for cyclic spectral analysis of weak signals in order to counteract the cycle leakage phenomenon [7, Chap. 13], [9, Secs. 5.3.4, 9.7] that is, the leakage at each cycle frequency coming from cyclic features of the signal-of-interest itself, the noise, and other interfering signals.

A zoom FAM algorithm based on the CZT is presented in [15] where, unlike the proposed ZSSCA, the points on the grid in the (α, ν) plane must respect several constraints and, as the original FAM, the cycle-frequency resolution is not constant in the analyzed region [5], [11].

As an example of application, the cyclic spectral analysis of a weak covert communication signal in the presence of strong noise and interference is carried out.

The paper is organized as follows. In Section II, notation and definitions on cyclic spectral analysis are introduced. The proposed ZSSCA is presented in Section III. Numerical results are shown in Section IV and conclusions are drawn in Section V.

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II. Cyclic Spectral Analysis

A discrete-time complex-valued signal $x(n)$ is said to be almost-cyclostationary in the wide-sense if its mean, autocorrelation, and conjugate autocorrelation functions are almost-periodic functions of time. Specifically, for the (conjugate) autocorrelation function one has

$$\mathrm{E}\left\{x(n+m)\, x^{(*)}(n)\right\} = \sum_{\alpha\in\mathcal{A}} R^{\alpha}_{xx^{(*)}}(m)\, e^{j2\pi\alpha n} \qquad (2.1)$$

where $\mathcal{A}$ is a countable set of possibly incommensurate frequencies, named *cycle frequencies*, and the Fourier coefficients

$$R^{\alpha}_{xx^{(*)}}(m) \triangleq \lim_{N\to\infty} \frac{1}{N} \sum_{n=-\lfloor N/2\rfloor}^{\lfloor N/2\rfloor} \mathrm{E}\left\{x(n+m)\, x^{(*)}(n)\right\} e^{-j2\pi\alpha n} \qquad (2.2)$$

are referred to as *(conjugate) cyclic autocorrelation functions* [7, Chap. 10], [9, Chap. 1]. In (2.1) and (2.2), $(*)$ denotes an optional complex conjugation which is present for the autocorrelation and is absent for the conjugate autocorrelation. The operator $\mathrm{E}\{\cdot\}$ has twofold interpretation. It is the statistical expectation in the classical stochastic approach where $x(n)$ is modeled as stochastic process and is the almost-periodic component extraction operator in the fraction-of-time probability approach where $x(n)$ is modeled as single function of time [7, Chap. 15], [9, Chap. 2]. In the latter case, $\mathrm{E}\{\cdot\}$ is redundant in (2.2).

The Fourier transform

$$S^{\alpha}_{xx^{(*)}}(\nu) = \sum_{m\in\mathbb{Z}} R^{\alpha}_{xx^{(*)}}(m)\, e^{-j2\pi\nu m} \qquad (2.3)$$

is referred to as the *(conjugate) cyclic spectrum* or *SCD*. It is the correlation between spectral components of $x(n)$ whose frequency separation is equal to α [7, Chap. 10], [9, Sec. 1.2.1.2].

The (conjugate) SCD can be estimated by the frequency-smoothed (conjugate) cyclic periodogram

$$S^{(N,\Delta\nu)}_{xx^{(*)}}(\alpha,\nu) \triangleq \left[\frac{1}{N}\, X_N(\nu)\, X_N^{(*)}((-)(\alpha-\nu))\right] \underset{\nu}{\otimes} H_{\Delta\nu}(\nu) \qquad (2.4)$$

where

$$X_N(\nu) \triangleq \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi\nu n} \qquad (2.5)$$

$H_{\Delta\nu}(\nu)$ is a smoothing window with bandwidth $\Delta\nu$, and $\otimes_\nu$ denotes convolution in the variable ν. In (2.4), $(-)$ is an optional minus sign linked to $(*)$. Under mild assumptions, the frequency-smoothed (conjugate) cyclic periodogram is consistent when $N\to\infty$ and $\Delta\nu\to 0$ with $N\Delta\nu\to\infty$. In practice, the condition $\Delta\nu \gg 1/N$ must be satisfied for estimate reliability [7, Chap. 11, Sec. B], [9, Sec. 5.4.4]

III. Zoom Cyclic Spectral Analysis

In this section, the proposed zoom strip spectral correlation analyzer is presented.

A. The Chirp Zeta Transform

The CZT [10] allows one to efficiently compute the zeta transform of a signal over a set of points in the complex plane by exploiting the FFT.

Let us consider the finite-length sequence $s(n)$, $n = 0,1,\dots,N-1$. By letting

$$z_k \triangleq A\, w^{-k} \qquad k=0,1,\dots,K-1, \qquad A\in\mathbb{C},\ w\in\mathbb{C} \qquad (3.1)$$

and using the identity $nk = [n^2+k^2-(k-n)^2]/2$, the following expression for the zeta transform of $s(n)$ at the points z_k of the complex plane is obtained

$$S(z_k) = \sum_{n=0}^{N-1} s(n)\, z_k^{-n} = w^{k^2/2} \sum_{n=0}^{N-1} \left[s(n)\, A^{-n}\, w^{n^2/2}\right] w^{-(k-n)^2/2}. \qquad (3.2)$$

The function $S(z_k)$ is called the CZT of $s(n)$ [10]. Since the sum in the right-hand side of (3.2) is a convolution, the CZT $S(z_k)$ is efficiently implemented by FFT.

By taking

$$A = e^{j2\pi\alpha_a} \qquad w = e^{-j2\pi(\alpha_b-\alpha_a)/K} \qquad (3.3)$$

the points z_k lie on an arc of the unit circle and $S(z_k)$, which in the following is denoted as $\mathrm{CZT}[s(n)](\alpha_k)$, is the Fourier transform of $s(n)$ in the frequency interval $[\alpha_a,\alpha_b)$ at the frequency bins

$$\alpha_k = \alpha_a + k\,(\alpha_b-\alpha_a)/K, \quad k=0,\dots,K-1. \qquad (3.4)$$

This property is exploited in the ZSSCA algorithm to zoom the SCD estimate in the cycle-frequency domain.

B. Zoom SSCA

Let $h_M(n)$ be an M-duration, even data-tapering window. Thus, the sequence $h_M(n)\, e^{j2\pi\nu n}$ is the impulse response function of a band-pass filter with central frequency ν and bandwidth $\Delta\nu = 1/M$.

The signal $x(n)$ is channelized into narrow-band components centered at frequencies ν with bandwidth $\Delta\nu$ by computing the convolution

$$\widetilde{X}_{\Delta\nu}(n,\nu) \triangleq \sum_{m=-M/2}^{M/2-1} \left[h_M(m)\, e^{j2\pi\nu m}\right] x(n-m) \qquad (3.5)$$

where, for the sake of simplicity, M is assumed to be even.

Let $g(n)$ be a real-valued data-tapering window. By computing the CZT of $\widetilde{X}_{\Delta\nu}(n,\nu)\, x^{(*)}(n)\, g(n)$ with parameters (3.3) one obtains

$$\frac{1}{N}\,\mathrm{CZT}\left[\widetilde{X}_{\Delta\nu}(n,\nu)\, x^{(*)}(n)\, g(n)\right](\alpha_k) = \frac{1}{N}\,\mathrm{CZT}\left[\sum_{m=-M/2}^{M/2-1} h_M(m)\, e^{j2\pi\nu m}\, x(n-m)\; x^{(*)}(n)\, g(n)\right](\alpha_k)$$

$$= \sum_{m'=-M/2}^{M/2-1} h_M(-m')\, e^{-j2\pi\nu m'}$$

$$\cdot \frac{1}{N}\, \mathrm{CZT}\left[x(n+m')\, x^{(*)}(n)\, g(n) \right](\alpha_k)$$

$$= \sum_{m'=-M/2}^{M/2-1} h_M(m')\, e^{-j2\pi\nu m'}\, R^{(N)}_{xx^{(*)}}(\alpha_k, m')\,. \qquad (3.6)$$

In (3.6),

$$R^{(N)}_{xx^{(*)}}(\alpha_k, m) \triangleq \frac{1}{N} \sum_{n=0}^{N-1} x(n+m)\, x^{(*)}(n)\, g(n)\, e^{-j2\pi\alpha_k n} \qquad (3.7)$$

with α_k in (3.4), is the discrete-time (conjugate) cyclic correlogram with data-tapering window $g(n)$. In (3.6), in the second equality, the variable change $m' = -m$ is made and in the third equality $h_M(m') = h_M(-m')$ is used since $h_M(\cdot)$ is an even sequence.

The (conjugate) cyclic correlogram (3.7) is the inverse Fourier transform of the (conjugate) cyclic periodogram [7, Chap. 11, Sec. A], [9, Lemma 2.39]

$$I^{(N)}_{xx^{(*)}}(\alpha_k, \nu) \triangleq \frac{1}{N}\, X_N(\nu)\, X^{(*)}_{g,N}((-)(\alpha_k - \nu)) \qquad (3.8)$$

where $X_{g,N}(\nu) \triangleq X_N(\nu) \otimes G(\nu)$ is the finite Fourier transform of the tapered signal $x(n)\, g(n)$, $n = 0, \dots, N-1$, with $G(\nu)$ Fourier transform of $g(n)$.

Denoting by $H_{\Delta\nu}(\nu)$ the Fourier transform of $h_M(n)$ and using the dual of the convolution theorem for Fourier transforms, from (3.6) one has (Fig. 1)

$$\frac{1}{N}\, \mathrm{CZT}\left[\widetilde{X}_{\Delta\nu}(n,\nu)\, x^{(*)}(n)\, g(n) \right](\alpha_k)$$

$$= I^{(N)}_{xx^{(*)}}(\alpha_k, \nu) \underset{\nu}{\otimes} H_{\Delta\nu}(\nu)$$

$$= S^{(N,\Delta\nu)}_{xx^{(*)}}(\alpha_k, \nu) \qquad (3.9)$$

with α_k in (3.4). The right-hand side of (3.9) is the frequency-smoothed (conjugate) cyclic periodogram and the left-hand side is the proposed ZSSCA.

The harmonic response $H_{\Delta\nu}(\nu)$ is low-pass with bandwidth $\Delta\nu = 1/M$. Thus, $\Delta\nu$ is the spectral-frequency resolution. If the spectral-frequency interval of interest is $[\nu_a, \nu_b)$, then the spectral-frequency samples

$$\nu_p = \nu_a + p\, \Delta\nu, \quad p = 0, 1, \dots, P-1$$

with $P = \lfloor (\nu_b - \nu_a)/\Delta\nu \rfloor$, are sufficient to describe $S^{(N,\Delta\nu)}_{xx^{(*)}}(\alpha_k, \nu)$.

For the frequency-smoothed (conjugate) cyclic periodogram computed on a data-record length N, the cycle-frequency resolution is $\Delta\alpha = 1/N$ [7, Chap. 11, Sec. B], [9, Sec. 5.2.3]. Thus, the value of K to not skip any cycle frequency in the cycle frequency interval $[\alpha_a, \alpha_b)$ must be large enough so that the cycle-frequency step $(\alpha_b - \alpha_a)/K$ of the CZT in (3.9) is smaller than the cycle-frequency resolution $\Delta\alpha = 1/N$. Larger values of K provide a better interpolation in the cycle-frequency domain.

When $\alpha_a = -\alpha_b = 1/2$ and $\nu_a = -\nu_b = 1/2$, the ZSSCA covers the whole cycle- and spectral-frequency domain and reduces to a modified version of the originally proposed SSCA [4], [5], [11]. The modification consists in considering strips in the (α, ν)-plane parallel to the ν axis rather than parallel to the diagonal line $\alpha = -2\nu$. Such a modification is due to the adopted asymmetric definition of (conjugate) cyclic autocorrelation (2.2) and cyclic spectrum (2.3) rather than the symmetric one [9, Sec. 1.2.1]. Thus, the principal domain in the (α, ν)-plane is a square rather than a diamond as in [4], [5], [11]. As a consequence, for the zoom SSCA proposed in this paper, the bins of the grid in the (α, ν)-plane can be chosen independently along α and ν, only accounting for the required cycle- and spectral-frequency resolutions which are constant in the whole analysis domain.

In contrast, for the zoom FAM [15], the points on the grid must respect several constraints in order to obtain the desired point spacing and the spectral-frequency resolution is not constant similarly to the original FAM.

C. Memory Occupancy

The matrix with the SCD estimates by the ZSSCA with cycle-frequency resolution $\Delta\alpha = 1/N$ and spectral-frequency resolution $\Delta\nu$ at points (α_k, ν_p), $k = 0, 1, \dots, K-1$ and $p = 0, 1, \dots, P-1$, has size $K \times P$. In contrast, the matrix with the SCD estimates by the SSCA with the same cycle- and spectral-frequency resolution in the whole (α, ν) domain has size $N \times N'$, where $N' = \lfloor \Delta\nu \rfloor$. Since in practical applications $K \ll N$ and $P < N'$, the proposed ZSSCA provides a significant computational advantage with respect to first performing the SCD estimation in the whole (α, ν) domain and then extracting the desired portion of SCD estimate. In addition, when $(\alpha_b - \alpha_a) \ll 1$, a finer interpolation in the cycle-frequency domain can be obtained by increasing the value of K in the ZSSCA rather than performing a zero padding as it would be required in the SSCA with consequent increase of memory occupancy and computational cost. Such a computationally-cheap finer interpolation allows one to get a better accuracy in cycle frequency estimation.

IV. Numerical Results

In this section, the cyclic spectral analysis of a weak covert direct-sequence spread-spectrum (DSSS) signal in the presence of a strong pulse-amplitude modulated (PAM) signal and circular white Gaussian noise (CWGN) (white in the band $[-f_s/2, f_s/2]$, with $f_s = 1/T_s$ sampling frequency) is carried out.

The covert signal is a short-code DSSS signal with binary white modulating sequence, binary spreading code with $N_c = 32$ chip per bit, and chip period $T_c = 7\, T_s$ which modulates a carrier residual at frequency $\nu_r f_s$, with $\nu_r = 0.00001$. It exhibits cyclostationarity at cycle frequencies $k/(N_c T_c)$, with $k \in \mathbb{Z}$ [9, Sec. 7.4] and conjugate cyclostationarity at conjugate cycle frequencies $k/(N_c T_c) + 2\nu_r\, f_s$, with $k \in \mathbb{Z}$

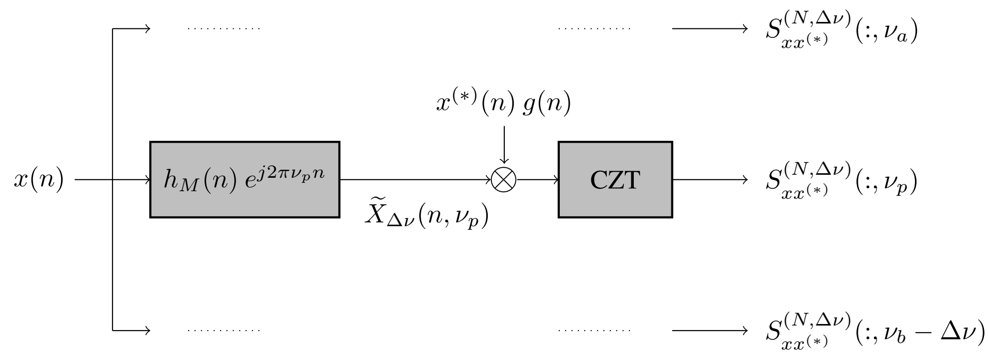

Fig. 1: Zoom strip spectral correlation analyzer

[9, Example 3.11]. The discrete-time sampled DSSS signal is almost cyclostationary with cycle frequencies $kT_s/(N_cT_c)$, $k = 0, 1, \cdots, \lfloor N_cT_c/T_s \rfloor - 1$ [9, Sec. 3.6.2] and conjugate cycle frequencies $(kT_s/(N_cT_c) + 2\nu_r)$ modulo 1, $k \in \mathbb{Z}$ [9, Corollary 3.18]. The PAM signal has binary white modulating sequence, full duty cycle rectangular pulse, and bit period $T_p = 16\, T_s$. It exhibits cyclostationarity and conjugate cyclostationarity at cycle frequencies k/T_p, with $k \in \mathbb{Z}$ [9, Sec. 7.3]. The discrete-time sampled PAM signal is cyclostationary with (conjugate) cycle frequencies kT_s/T_p, $k = 0, 1, \cdots, T_p/T_s - 1$ [9, Sec. 3.6.2].

Let $P_{\rm PAM}$, $P_{\rm DSSS}$, and $P_{\rm CWGN}$ be the average powers of the PAM, DSSS, and CWGN signals, respectively. The signal-to-noise ratio (SNR), defined as $\mathrm{SNR} = 10 \log(P_{\rm DSSS}/P_{\rm CWGN})$, is –8 dB, and the signal-to-interference ratio (SIR), defined as $\mathrm{SIR} = 10 \log(P_{\rm DSSS}/P_{\rm PAM})$, is –8 dB. The numerical bandwidths of the PAM and DSSS signals are $T_s/T_p = 1/16$ and $T_s/T_c = 1/7$, respectively. Therefore, the power spectral density of the DSSS signal is completely covert by that of the PAM and the CWGN signals (Fig. 2).

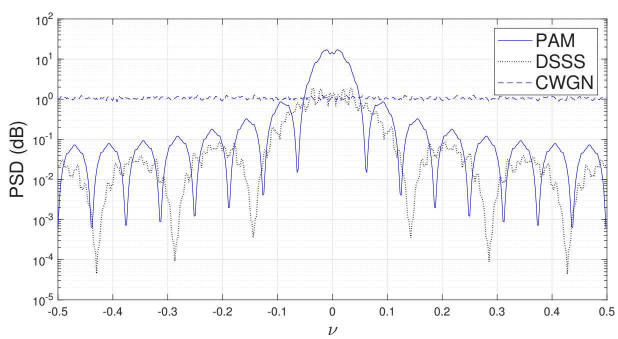

Fig. 2: Power spectral densities of the PAM, covert DSSS, and CWGN signals.

The cyclic spectral analysis of the discrete-time signal $x(n)$ is carried out by the SSCA and ZSSCA with optional complex conjugation $(*)$ present. The number of samples is $N = 2^{21}$ and the spectral-frequency resolution is $\Delta\nu = 1/2^9$ for both analyzers.

The ZSSCA estimates the cyclic spectrum in the rectangular region $[\alpha_a, \alpha_b) \times [\nu_a, \nu_b) = [0.5\, \alpha_{\rm DSSS}, 3.5\, \alpha_{\rm DSSS}) \times [-0.1, 0.1)$, where $\alpha_{\rm DSSS} = T_s/(N_cT_c) = 0.00446428571$ is the smallest (in magnitude) cycle frequency of the sampled DSSS signal.

The strength of the SCD defined as

$$\lambda^{(N,\Delta\nu)}_{xx^*}(\alpha) \triangleq \int_{\mathcal{B}} \left| S^{(N,\Delta\nu)}_{xx^*}(\alpha,\nu) \right|^2 \mathrm{d}\nu \qquad (4.1)$$

is also computed, where the integration interval $\mathcal{B}$ is the whole frequency interval $[-1/2, 1/2)$ for the SSCA and is $[\nu_a, \nu_b)$ for the ZSSCA.

In Fig. 3, (top) the magnitude of the SCD estimate by the SSCA in the whole (α, ν) domain $[-1/2, 1/2) \times [-1/2, 1/2)$ and (bottom) its strength are reported. The SCD of the PAM signal and a floor for $\alpha = 0$ corresponding to the CWGN are evident. In contrast, cyclic features of the covert DSSS signal cannot be recognized.

In Fig. 4, (top) the magnitude of the SCD estimate by the ZSSCA in the rectangular region $[\alpha_a, \alpha_b) \times [\nu_a, \nu_b)$ of the (α, ν) domain and (bottom) its strength are reported. The zoom in the cycle frequency domain highlights the SCD of the DSSS signal at the cycle frequencies $k\, \alpha_{\rm DSSS}$, $k = 1, 2, 3$.

Both SSCA and ZSSCA have cycle frequency resolution $1/N \simeq 4.77 \cdot 10^{-7}$. For the SSCA, the cycle-frequency step is coincident with the cycle-frequency resolution, and the number of channels is $N' = \lfloor 1/\Delta\nu \rfloor = 2^9$. Thus, the size of the SCD matrix of the SSCA is $N \times N' = 2^{21} \times 2^9 \simeq 10^9$ elements. For the ZSSCA, $K = \lfloor 4N(\alpha_b - \alpha_a) \rfloor = \lfloor 12N\alpha_{\rm DSSS} \rfloor = 112347$ is such that the cycle-frequency step is $1/(4N)$. The number of channels is $P = \lfloor (\nu_b - \nu_a)/\Delta\nu \rfloor = 102$. Thus, the size of the SCD matrix for the ZSSCA is $K \times P = 112347 \times 102 \simeq 1.14 \cdot 10^7$.

Thus, if one is only interested in the cyclic spectral analysis in the rectangular region $[\alpha_a, \alpha_b) \times [\nu_a, \nu_b)$, it would be very impractical to first estimate the SCD matrix in the whole (α, ν) domain and then extract from this matrix the sub-matrix corresponding to the desired region.

The size of the SCD matrix of the ZSSCA is two orders of magnitude smaller than that of the SCD matrix obtained by the SSCA, and, moreover, the cycle-frequency step is 1/4 of that of the SSCA. In order to obtain the same cycle-frequency step

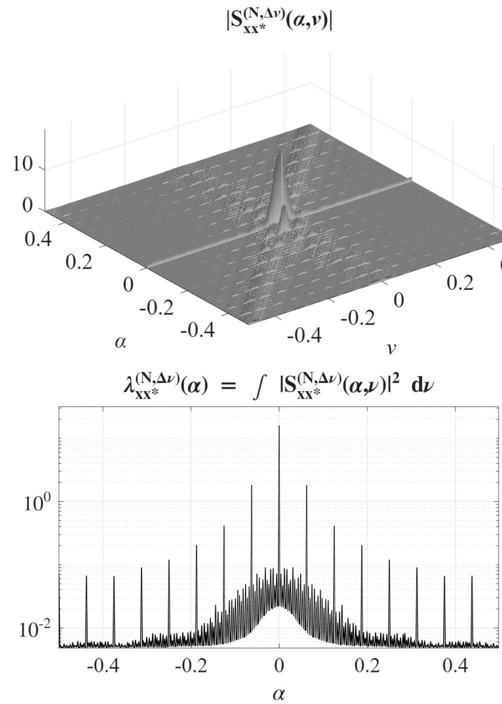

Fig. 3: SSCA: (Top) magnitude of the SCD estimate in the whole (α, ν) domain $[-1/2, 1/2) \times [-1/2, 1/2)$; (Bottom) strength of the SCD estimate.

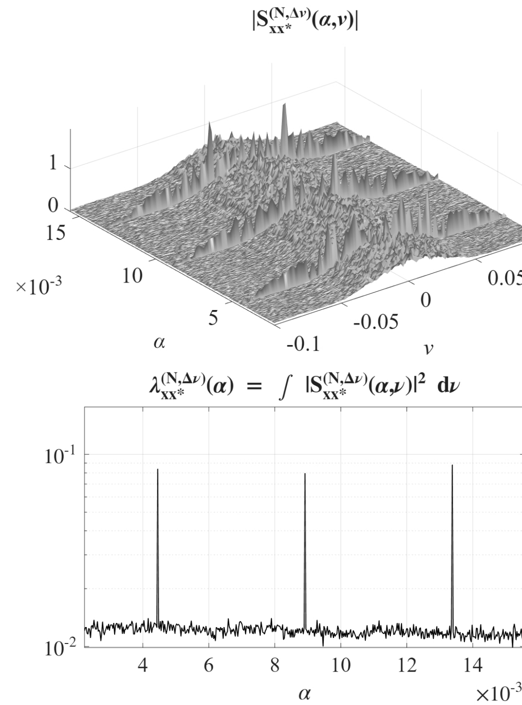

Fig. 4: Zoom SSCA: (Top) magnitude of the SCD estimate in the rectangular region $[\alpha_a, \alpha_b) \times [\nu_a, \nu_b)$ of the (α, ν) domain; (Bottom) strength of the SCD estimate.

with the SSCA, a zero padding factor of 4 should be adopted increasing of a factor 4 the size of the SCD matrix.

V. Conclusion

The novel zoom strip spectral correlation analyzer is proposed to estimate the SCD in a selected rectangular region of the cycle- and spectral-frequency plane. It exploits the CZT in a modified version of the SSCA in order to zoom in the cycle-frequency domain. The proposed analyzer has constant cycle- and spectral-frequency resolution. It is shown to be very parsimonious in memory occupancy with respect to extracting the desired estimated SCD sub-matrix from the SCD matrix estimated on the whole cycle- and spectral-frequency domain. As an example of application, the cyclic spectral analysis of a weak covert DSSS signal in the presence of a strong PAM signal and CWGN is carried out.

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